DAE-based Contracts

Team: Hycomes team, INRIA center at Rennes University
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About the Internship

In the field of systems engineering, **contract-based design** [1] is a modular methodology that enables independent component development while ensuring correct system-wide integration. A specific instance is the *assume–guarantee contract*: Contract $= (A, G)$. Here, **Assumptions** A describe what the component expects from its environment, while **Guarantees** G specify what the component promises to deliver, provided that the assumptions hold. Formally, a contract can be represented as an implication:

$$E \preccurlyeq A \Rightarrow \Sigma \wedge E \preccurlyeq G$$

meaning that if the environment satisfies the assumptions A, the system under the environment must ensure the guarantees G. This contract-based perspective supports *modular* and *compositional* system design:

- Each subsystem can be designed independently under its own contract.
- When composing systems, one can verify *compatibility* between contracts by checking that assumptions and guarantees are consistent.
- Contracts provide a *formal interface* for verifying correctness, safety, and performance across components.

In recent years, the design and analysis of large-scale *control systems* have become increasingly challenging. To address this, contract-based design has been introduced into the control systems domain. Two notable studies [2,3] develop contract frameworks for linear time-invariant (LTI) control systems:

$$\Sigma : \quad \begin{cases} \dot{x} = Ax + Bu, \\ y = Cx + Du. \end{cases}$$

In [2], the classical behavioral theory introduced by Jan Willems is used to formalize key contract-theoretic notions—such as assumptions, guarantees, refinement, and composition—for Σ. In [3], geometric control theory is employed to define simulation relations between two control systems, providing a foundation for implementing assume–guarantee contracts. A contract-based control design algorithm is then proposed based on these results.

The goal of this internship is to extend contract theory to **linear differential–algebraic equations (DAEs)**:

$$\Delta : \quad E\dot{z} = Az.$$

As a modular modeling approach derived from first-principle physics, DAEs frequently appear in constrained mechanical systems, power networks, and analog circuit design. Mathematically, DAEs offer several potential advantages for contract-based analysis:

1. System interconnections can be naturally expressed as algebraic equations, supporting a compositional framework.

2. DAEs treat all variables uniformly—states, inputs, and outputs—aligning well with the behavioral approach.

3. The geometric analysis of DAEs is well established [4] [5], providing effective tools for describing relations between systems and specifications.

[1] A. Benveniste, B. Caillaud, et all, *Contracts for System Design.* Foundations and Trends in Electronic Design Automation, Now Publishers, 2018.
[2] B. M. Shali, A. van der Schaft, and B. Besselink, "Composition of behavioural assume-guarantee contracts," *IEEE Transactions on Automatic Control*, pp. 1–16, 2022.
[3] B. M. Shali, A. van der Schaft, and B. Besselink, "Design and control for implementation of simulation-based assume-guarantee contracts," *IEEE Transactions on Automatic Control*, pp. 1–15, 2025.
[4] Y. Chen, and W. Respondek. "Geometric analysis of differential-algebraic equations via linear control theory." *SIAM Journal on Control and Optimization* 59.1 (2021): 103-130.
[5] F. L. Lewis "A survey of linear singular systems." *Circuits, systems and signal processing* 5.1 (1986): 3-36.

Responsibilities

- Conduct literature reviews on both contract theory and linear DAEs.
- Define notions from contract theory for DAE systems and develop theories on their verifications.
- Apply the proposed DAE-based contracts to simple examples
- Participate actively in team meetings and brainstorming sessions.

Requirements

- Master's student in systems and control, applied mathematics, computer science, or a related field.
- Proficiency in academic English writing and fluency in spoken English.
- Strong mathematical reasoning and problem-solving skills.
- Familiarity with one or more of the following topics is an advantage:
  DAEs, contract theory, geometric control

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