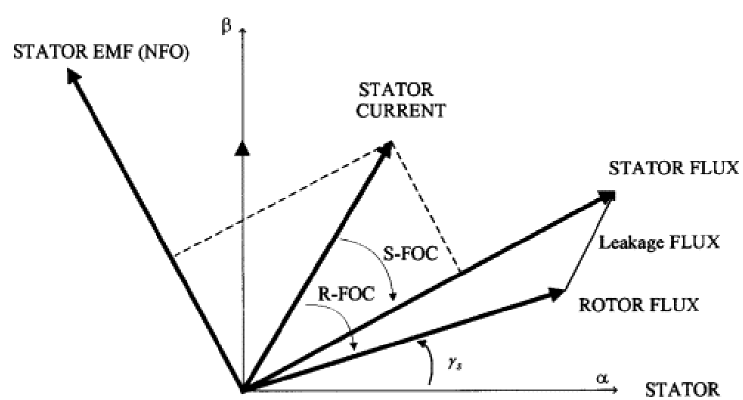
Why advanced control of electric motors?

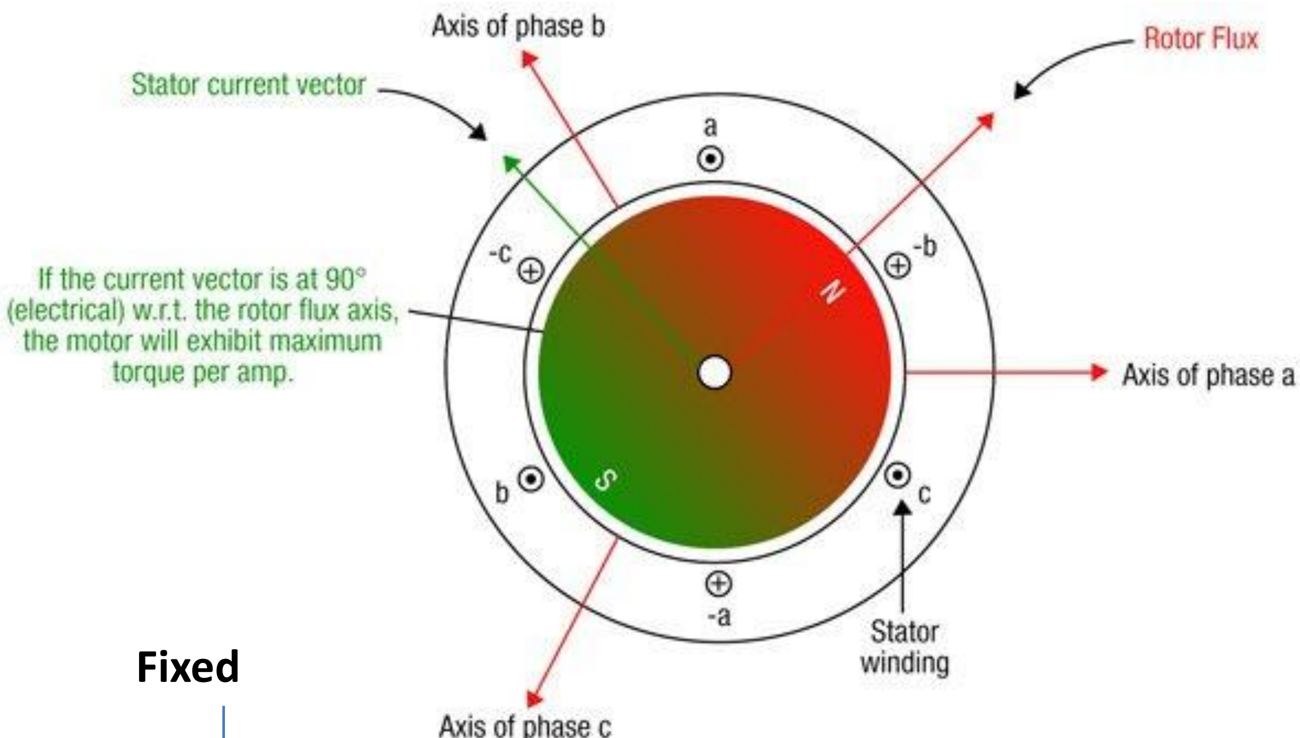
- The motors have the same topology since 1800
- The sychronous reluctance motor is invented in 1838
- The stepper motor is invented in 1930
- Development of digital system software and automatic control
- Develpoment of the control of electric motors
- Non modelled dynamics

Field Oriented Control

- The most exciting development in motor control
- Unified control topology that works with all motors
- The aim is to orient the motor fields
- Invented in 1968
- The control becomes very easy
- Cascade control topology



Field Oriented Control in real time



Constant

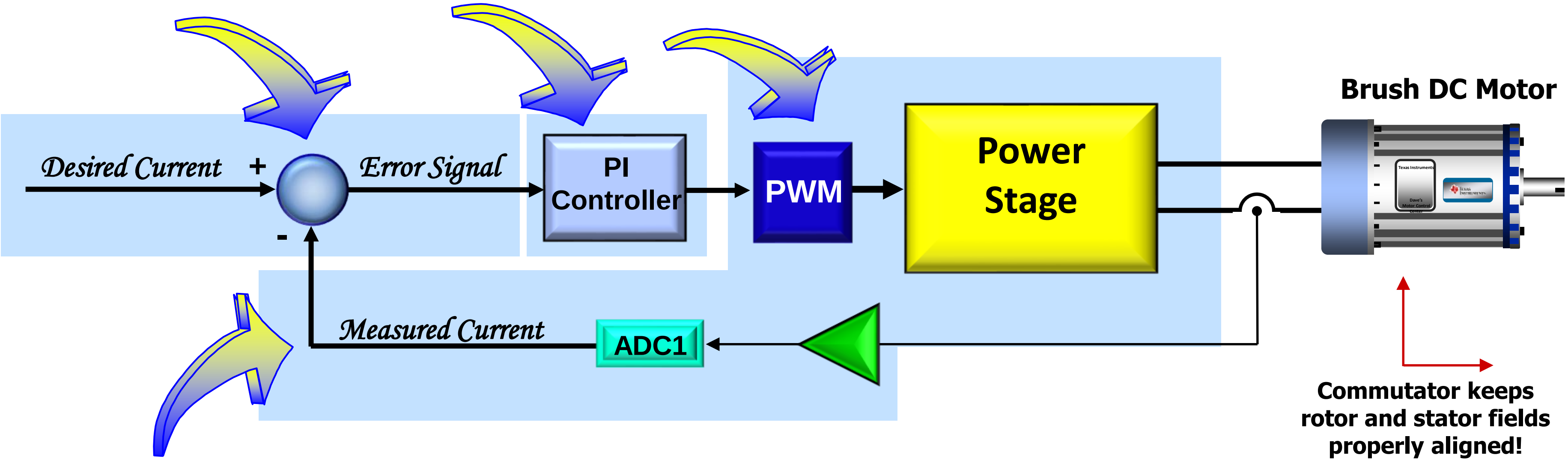
Fixed

$$T_e = \frac{3}{2}\frac{P}{2}\left[\lambda_{dr} I_{qs}\right]$$

Adjustable

If the phase between λ_{dr} and I_{qs} is not 90°?

How Do You Control Torque on a DC Motor?

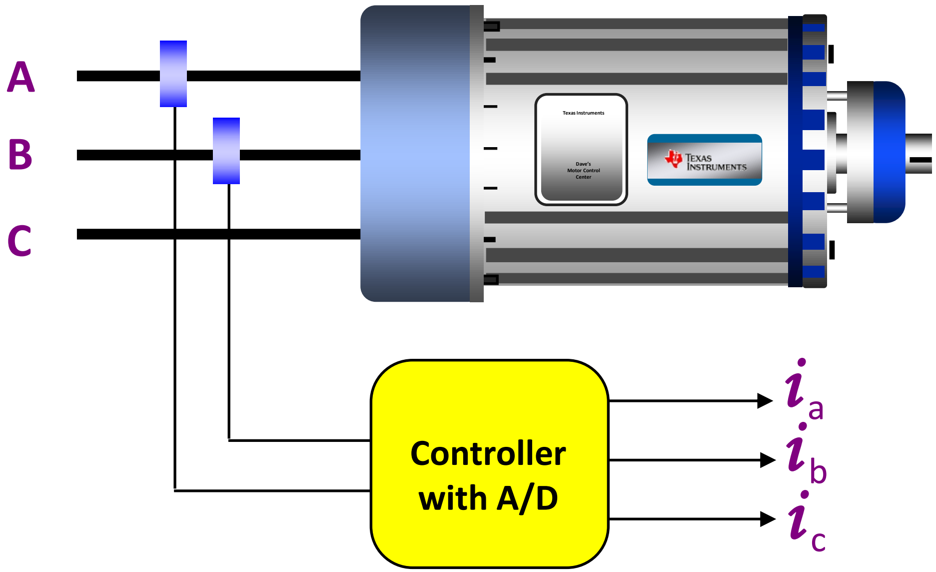


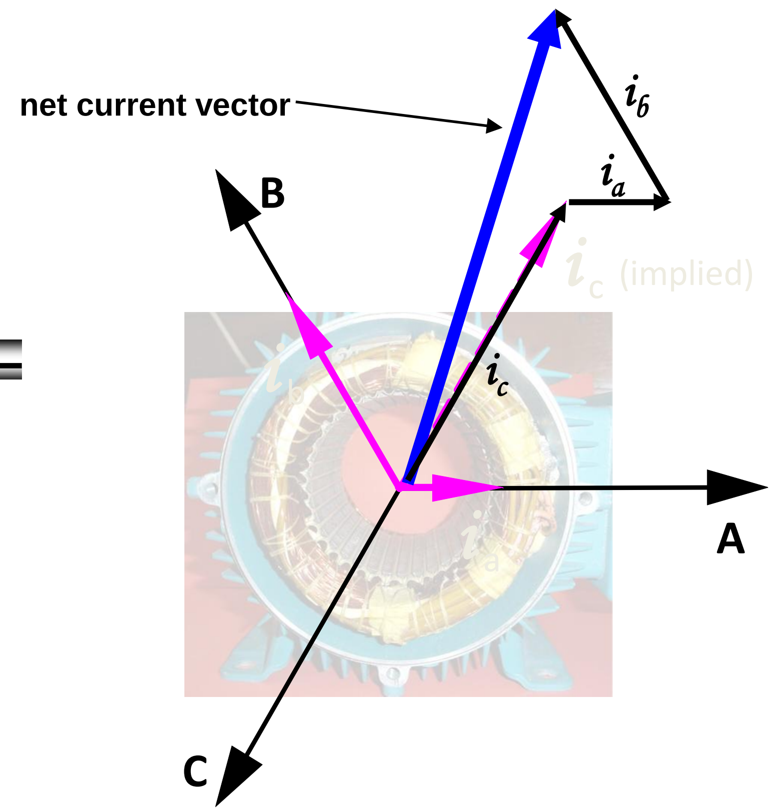
1. *Measure current already flowing in the motor.*
2. *Compare the measured current with the desired current, and generate an error signal.*
3. *Amplify the error signal to generate a correction voltage.*
4. *Modulate the correction voltage onto the motor terminals.*

$$Torque = K_a i$$

1. Measure currents already flowing in the motor.

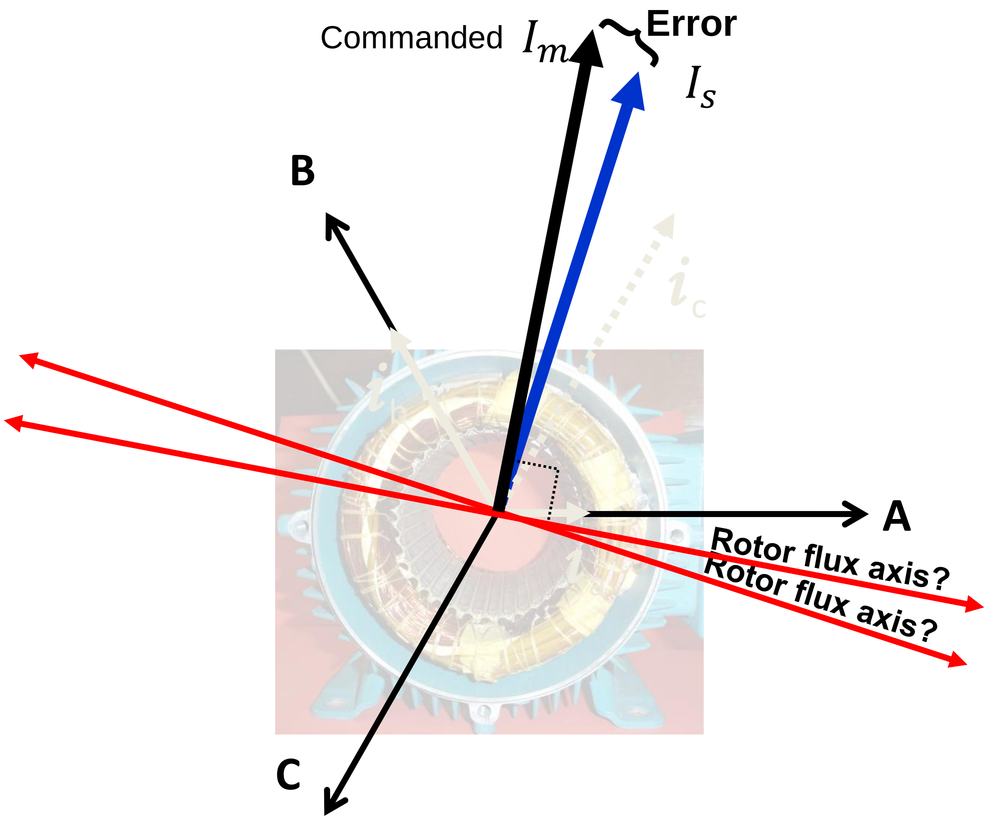
A, B, and C axes are “fixed” with respect to the motor housing. This reference frame is also called the “stationary frame” or “stator frame”.





2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

The desired phase currents can be calculated via these equations:



$$i_a = I_m \sin(\theta_\lambda)$$

$$i_b = I_m \sin(\theta_\lambda + 120^o)$$

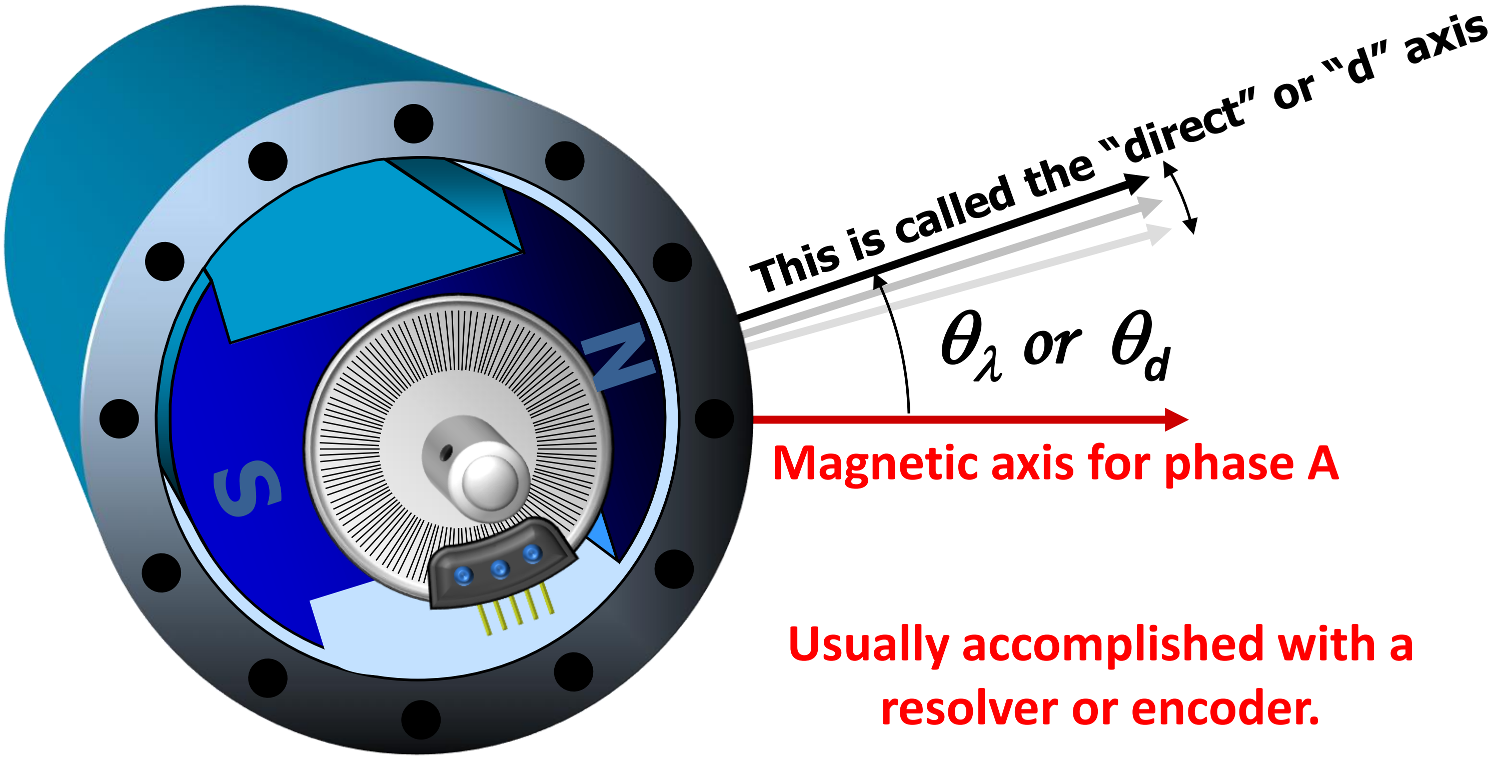
$$i_c = I_m \sin(\theta_\lambda - 120^o)$$

I_m ***is proportional to motor torque***

θ_λ ***is the angle of the rotor flux***

So how do we get the rotor flux angle?

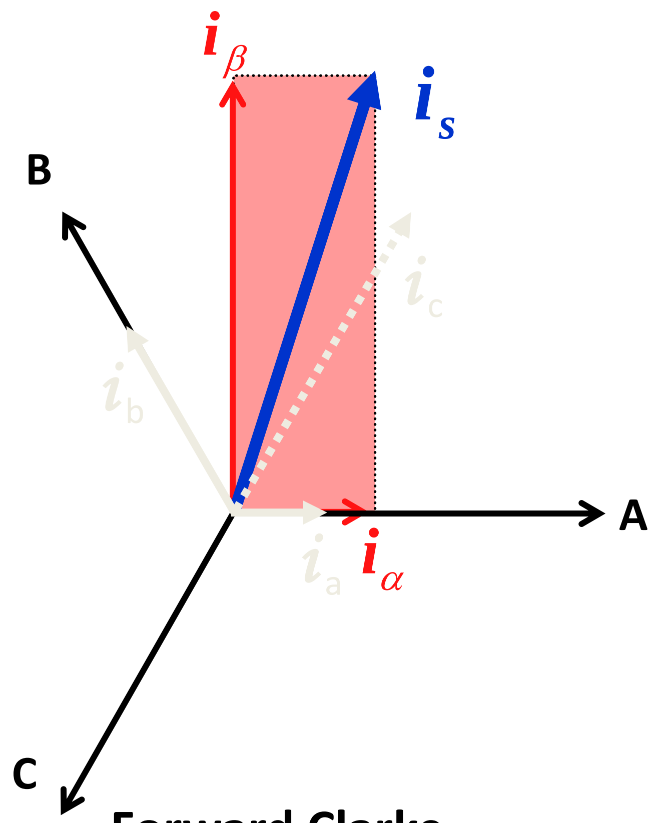
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.



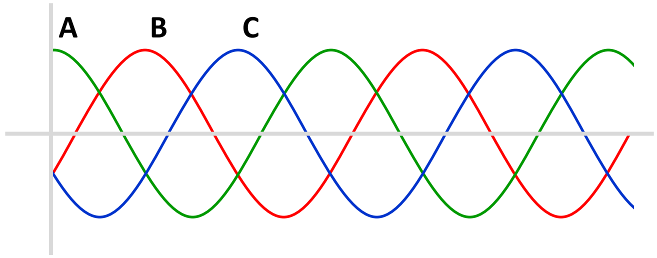
Usually accomplished with a resolver or encoder.

2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

The Concordia transform allows us to convert three vectors into two orthogonal vectors that produce the same net vector.



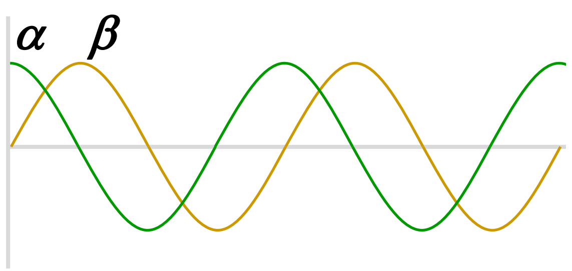
In other words, convert a 3-phase motor into a 2-phase motor.



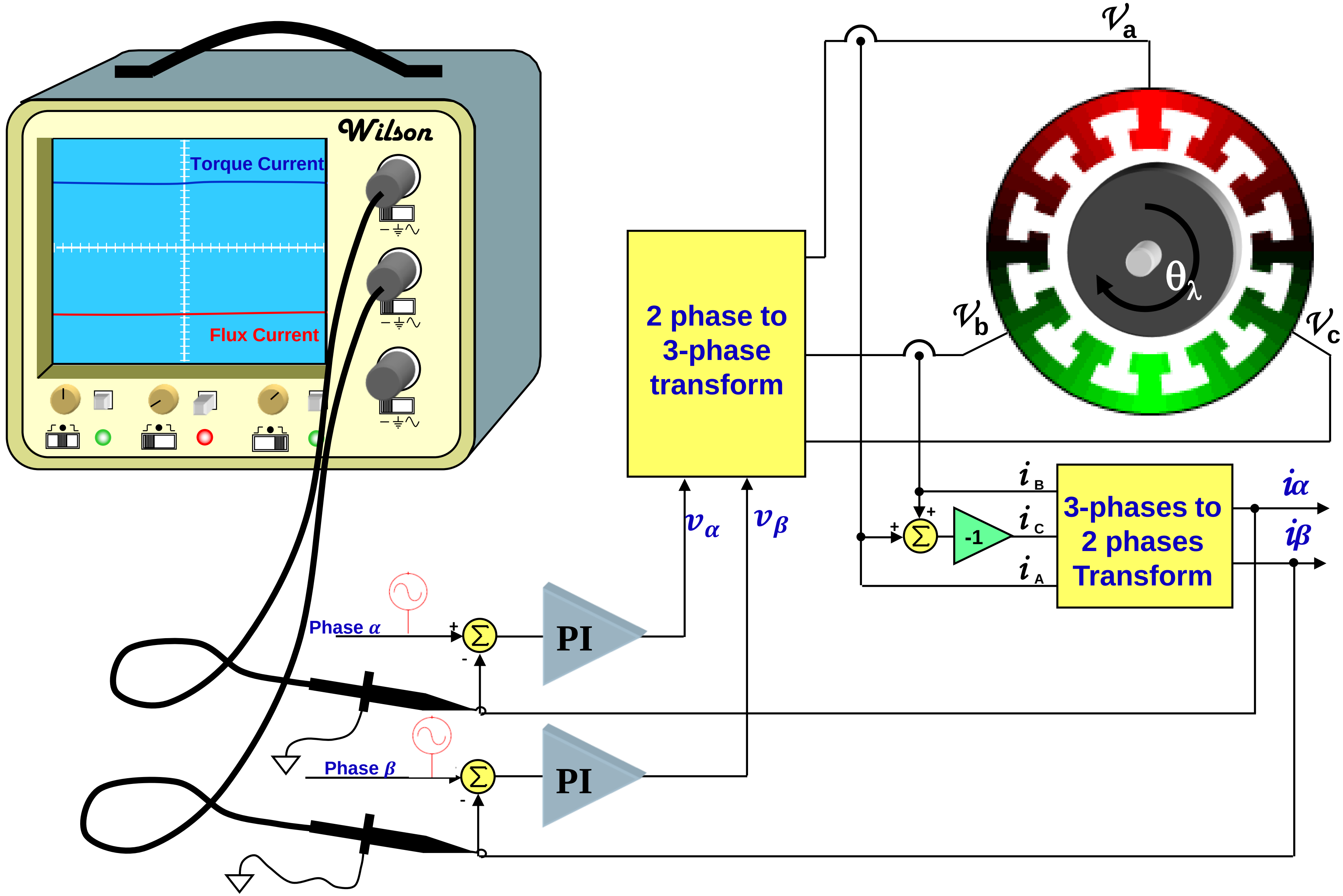
Forward Clarke

$$\alpha = A$$

$$\beta = \frac{(B - C)}{\sqrt{3}}$$

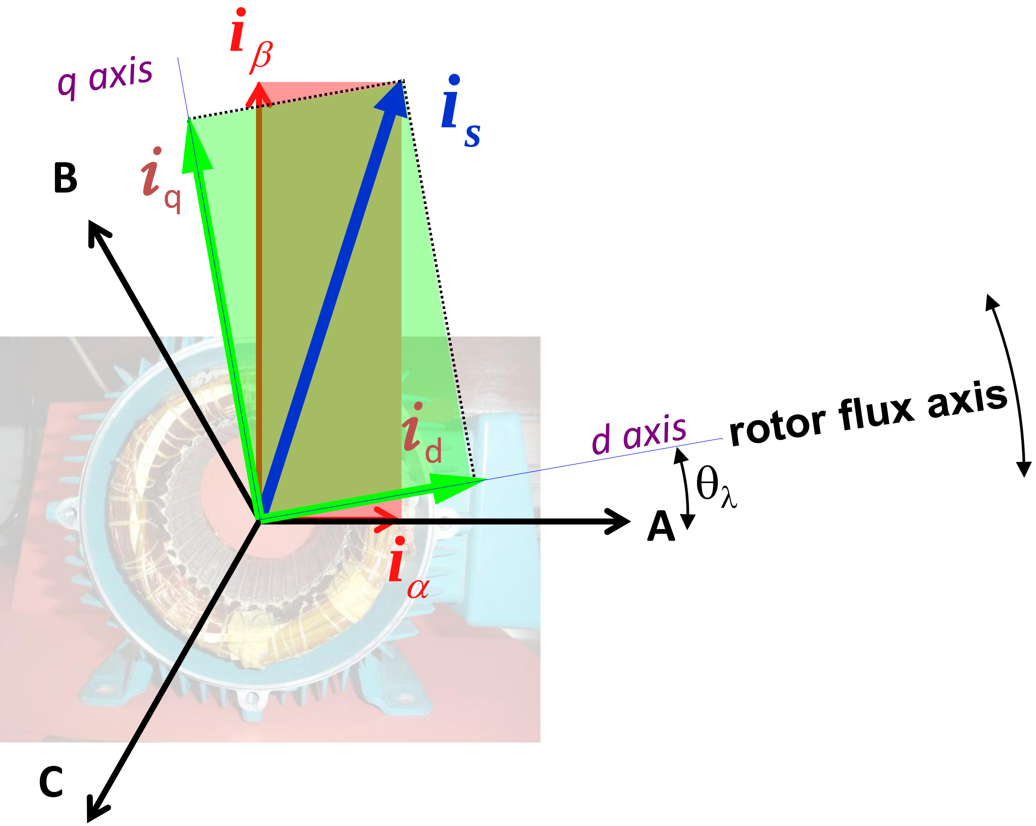


2-Phase stationary Frame Current Regulators

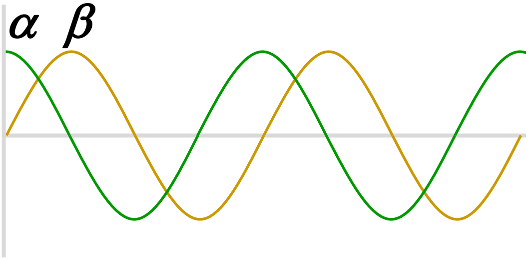


2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

Jump up on the rotating reference frame, whose x-axis is the rotor flux axis.

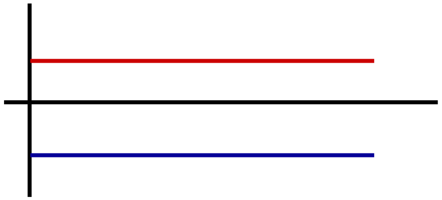


This is called the Park Transform



$$i_d = i_\alpha \cos\theta_\lambda + i_\beta \sin\theta_\lambda$$

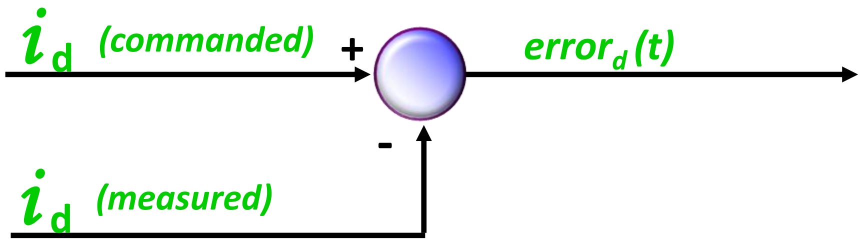
$$i_q = -i_\alpha \sin\theta_\lambda + i_\beta \cos\theta_\lambda$$



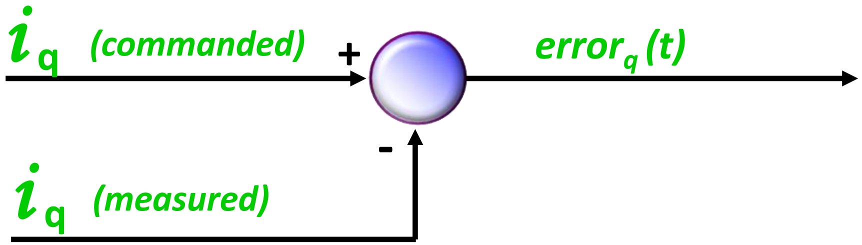
2. Compare the measured current (vector) with the desired current (vector), and generate error signals.

i_d and i_q are handled independently. Since the comparison is performed in the synchronous frame, motor AC frequency is not seen. Thus, they are DC quantities!

Under normal conditions, we have all the flux we need supplied by the permanent magnets on the rotor. So commanded i_d is set to zero.



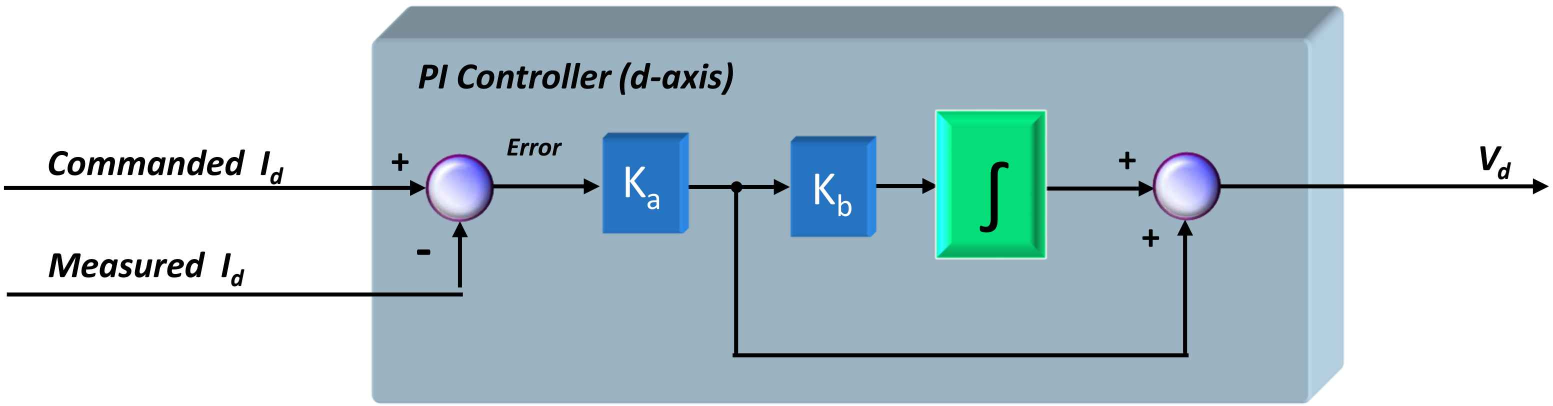
This is how much torque we want!

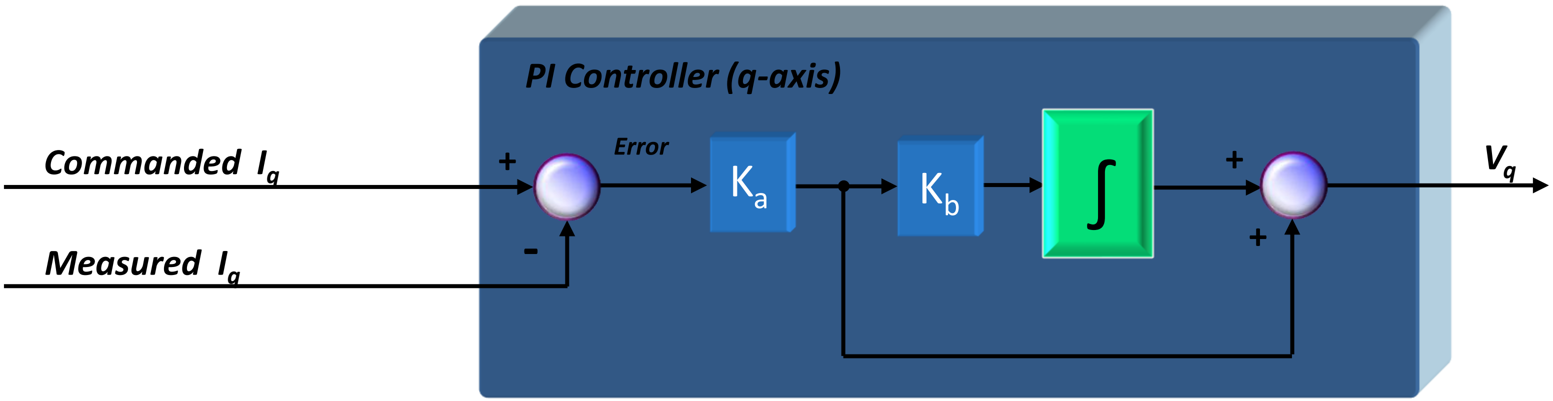


i_d can be used to weaken the field of the machine.

i_q controls the amount of torque generated by the motor

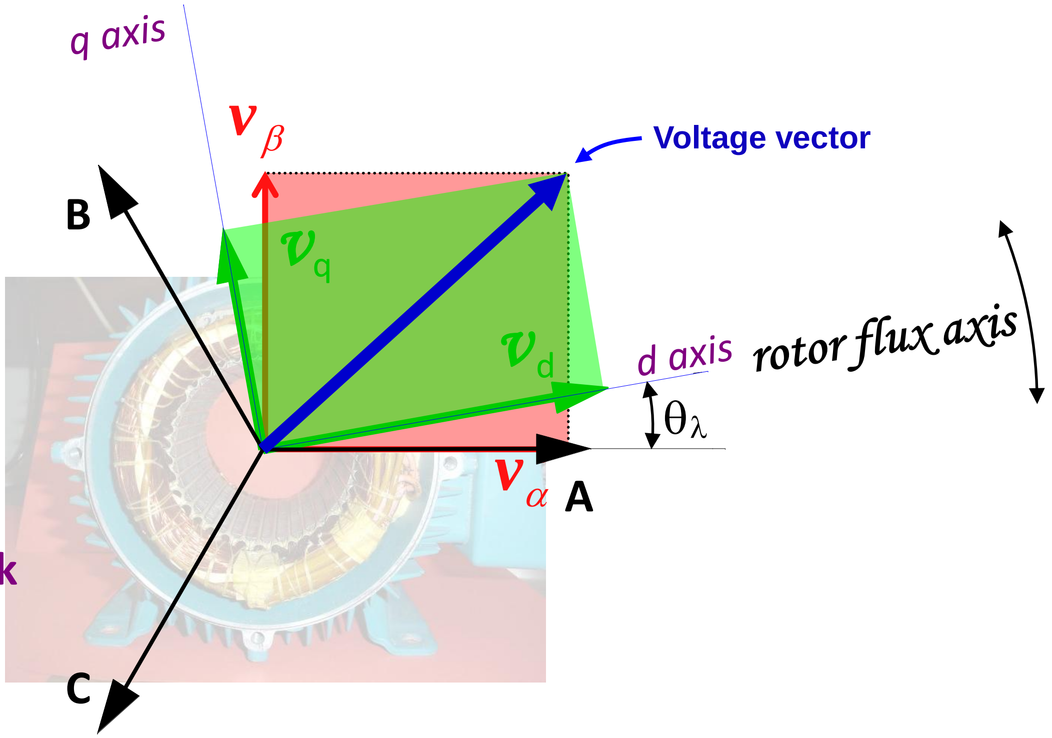
3. Amplify the error signals to generate correction voltages.



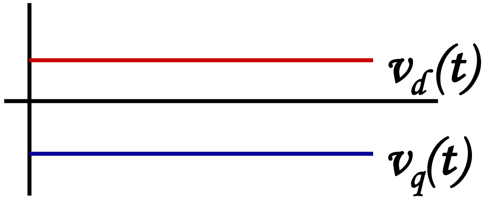


4. Modulate the correction voltages onto the motor terminals.

Before we can apply the voltages to the motor windings, we must first jump off of the rotating reference frame.

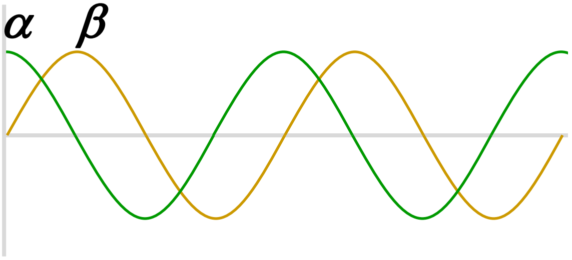


Part A. Transfer the voltage vectors back to the stationary rectangular coordinate system.



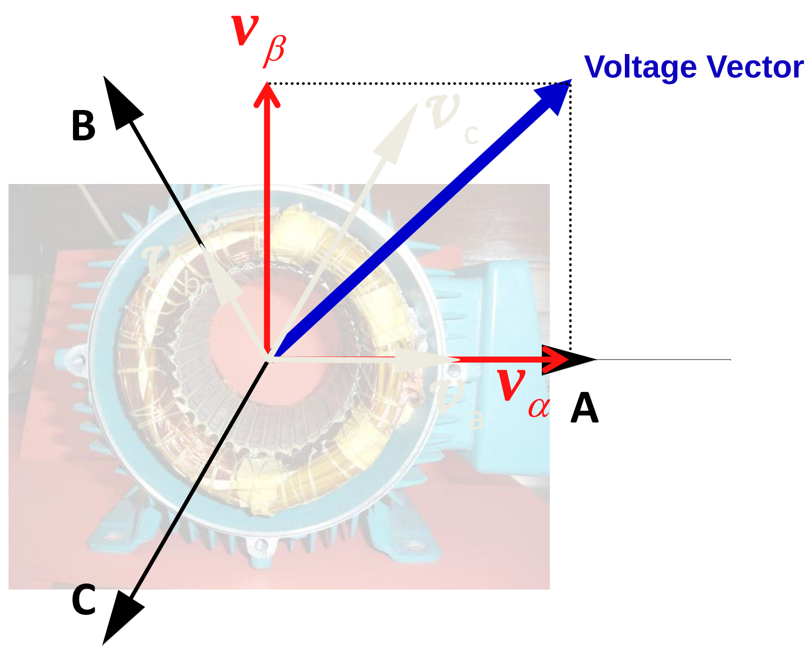
$$v_\alpha = v_d \cos\theta_\lambda - v_q \sin\theta_\lambda$$

$$v_\beta = v_d \sin\theta_\lambda + v_q \cos\theta_\lambda$$

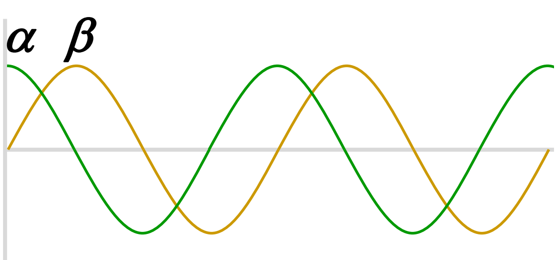


4. Modulate the correction voltages onto the motor terminals.

Part B. Next, we transform the voltage vectors from the rectangular coordinate system to three phase vectors.



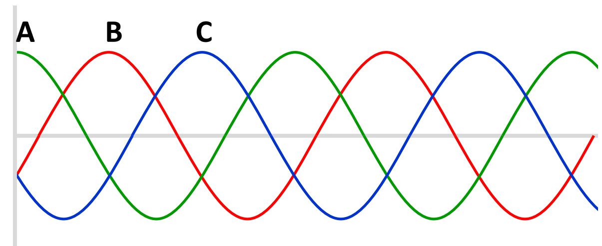
Reverse Clarke Transformation



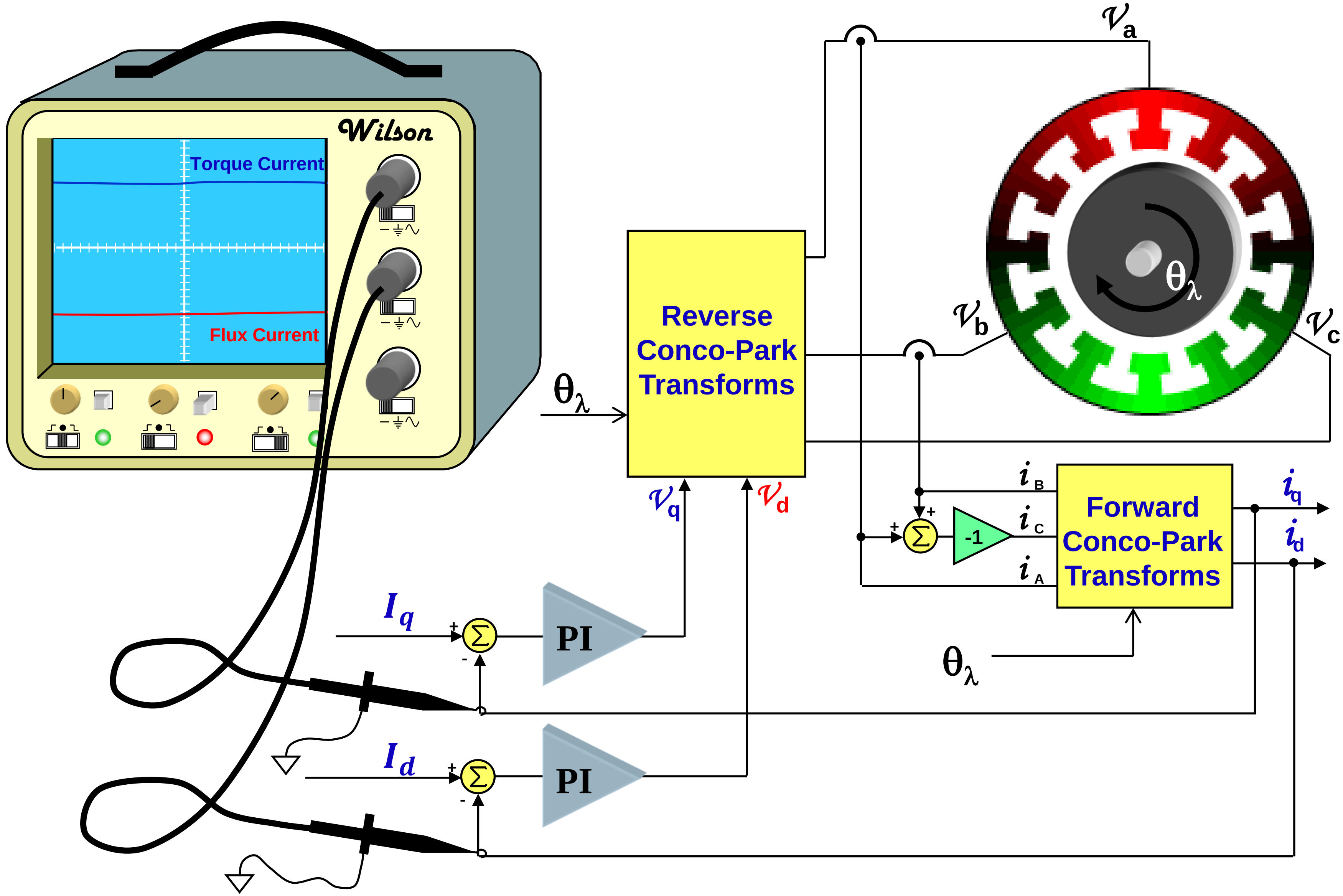
$$A = \alpha$$

$$B = -\tfrac{1}{2}\alpha + \tfrac{\sqrt{3}}{2}\beta$$

$$C = -\tfrac{1}{2}\alpha - \tfrac{\sqrt{3}}{2}\beta$$



FOC in rotating frame



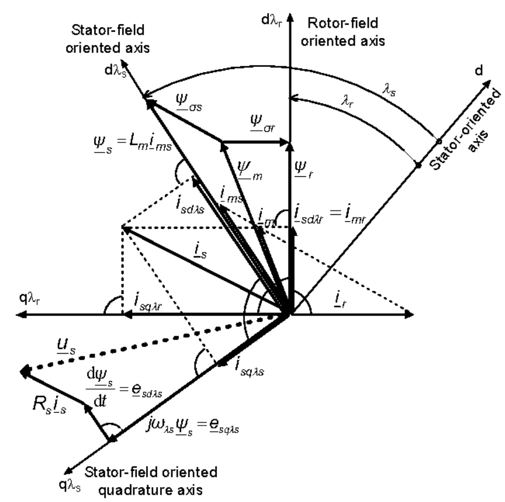
Scientific point of view

- Any model is used
- How the PI controllers are tuned?
- Is the stability analysed?
- Are the time varying uncertainties considered?

- Not acceptable from scientific point of view and for several industrial applications
- The machine model is needed for rigourous study
- Adavanced control strategies to compensate the lack of modeling and the uncertainties

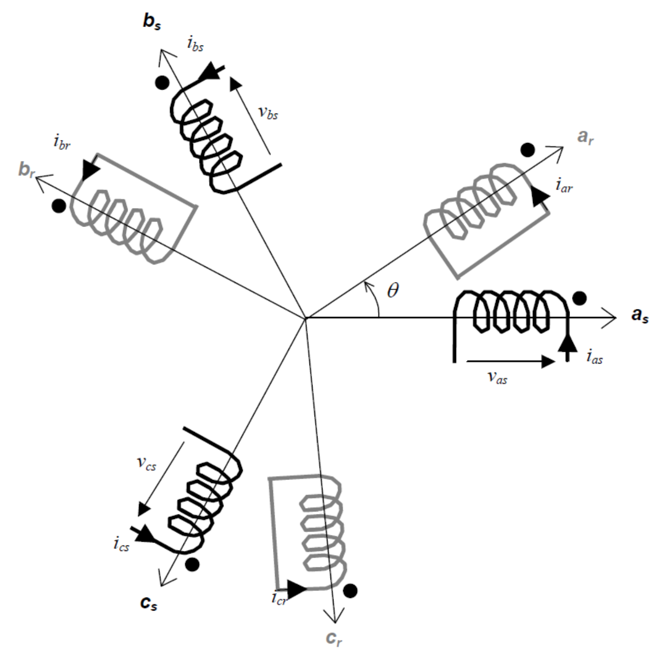
Field oriented control based on machine model

- Control the AC motors in same way as DC motors
- The task is no easier but more secure
- The problems related to the modelling and uncertainties is considered
- Stability could be proven
- Parameter tuning clear and easy to master
- The model is needed also for the estimation (position, speed, torque,…)



Modelling of AC machines

- Saturation and parameter changes are neglected
- Uniform air gap
- Uniformly distributed stator/rotor windings



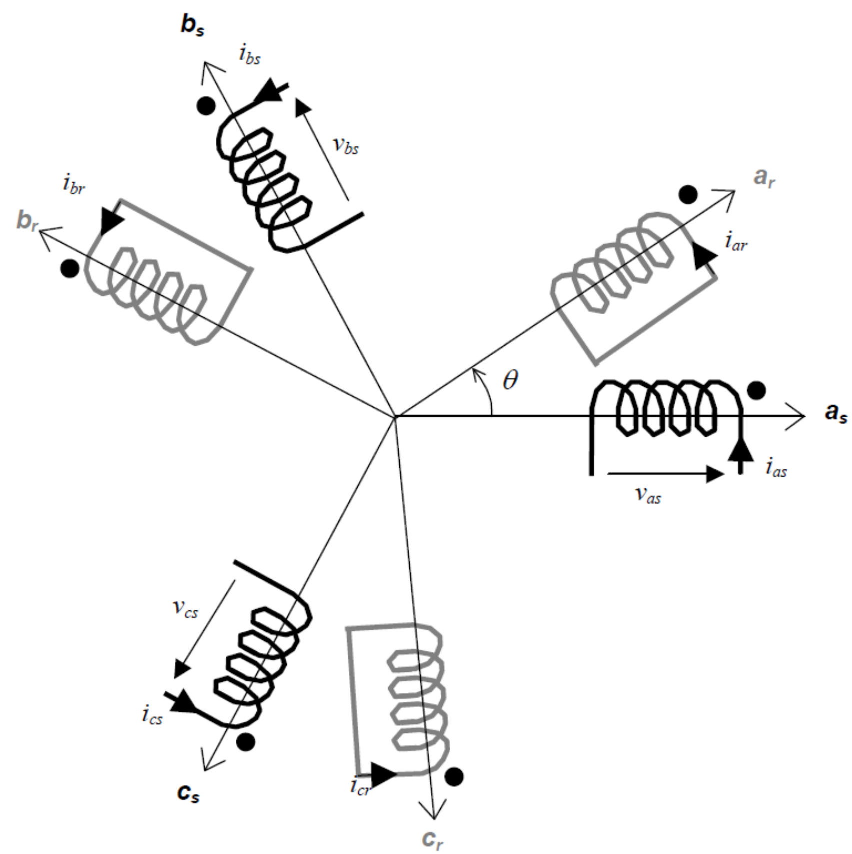
Modelling of Induction machine

Three phase voltages

$$\begin{cases} v_a = v_M \cos(\omega t) \\ v_b = v_M \cos(\omega t - \frac{2\pi}{3}) \\ v_c = v_M \cos(\omega t - \frac{4\pi}{3}) \end{cases}$$

Three phase currents

$$\begin{cases} I_a = I_M \cos(\omega t - \varphi) \\ I_b = I_M \cos(\omega t - \varphi - \frac{2\pi}{3}) \\ I_c = I_M \cos(\omega t - \varphi - \frac{4\pi}{3}) \end{cases}$$



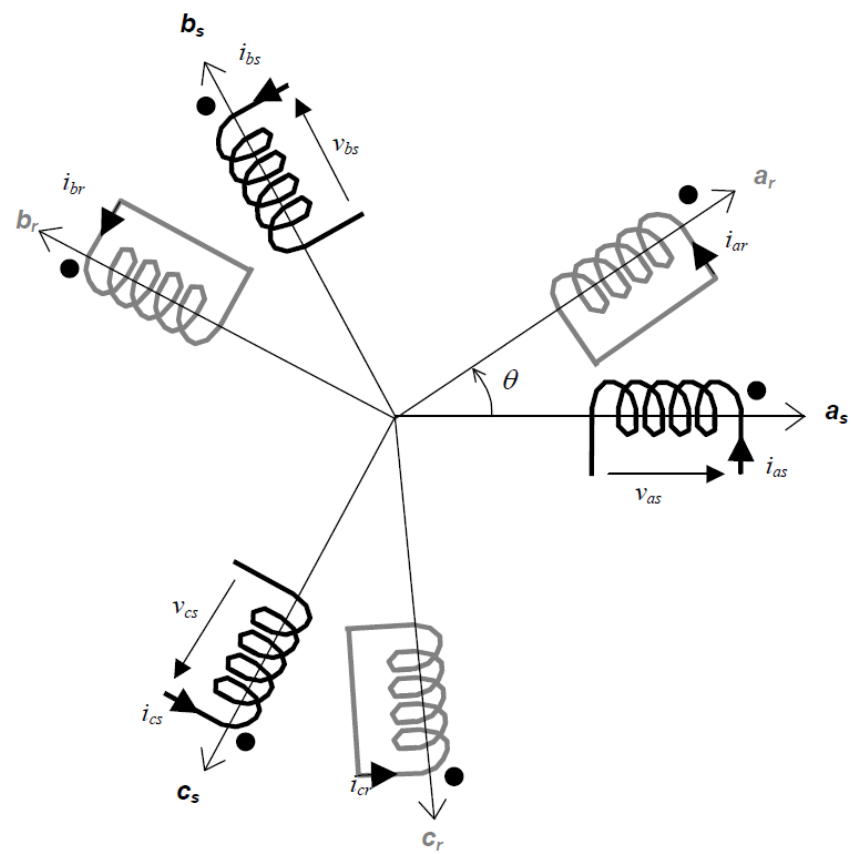
Modelling of Induction machine

Stator equations

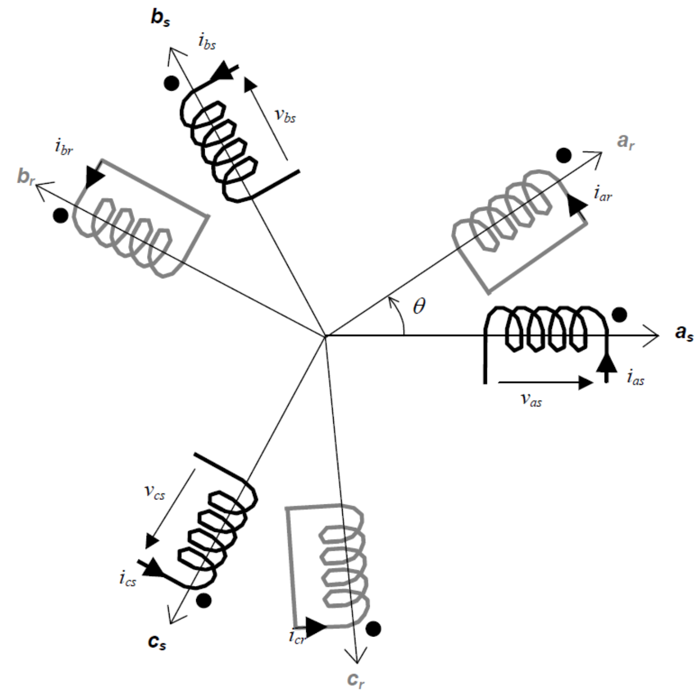
$$\begin{cases} v_{as} = R_s i_{as} + \dfrac{d\varphi_{as}}{dt} \\ v_{bs} = R_s i_{bs} + \dfrac{d\varphi_{bs}}{dt} \\ v_{cs} = R_s i_{cs} + \dfrac{d\varphi_{cs}}{dt} \end{cases}$$

Rotor equations

$$\begin{cases} v_{ar} = R_r i_{ar} + \dfrac{d\varphi_{ar}}{dt} \\ v_{br} = R_r i_{br} + \dfrac{d\varphi_{br}}{dt} \\ v_{cr} = R_r i_{cr} + \dfrac{d\varphi_{cr}}{dt} \end{cases}$$



Modelling of Induction machine



Flux equations

φ_{as} = Self flux + Mutual flux

φ_{as} = $L_s i_{as} + m_s i_{bs} + m_s i_{cs} + m_1 i_{ar} + m_3 i_{br} + m_2 i_{cr}$

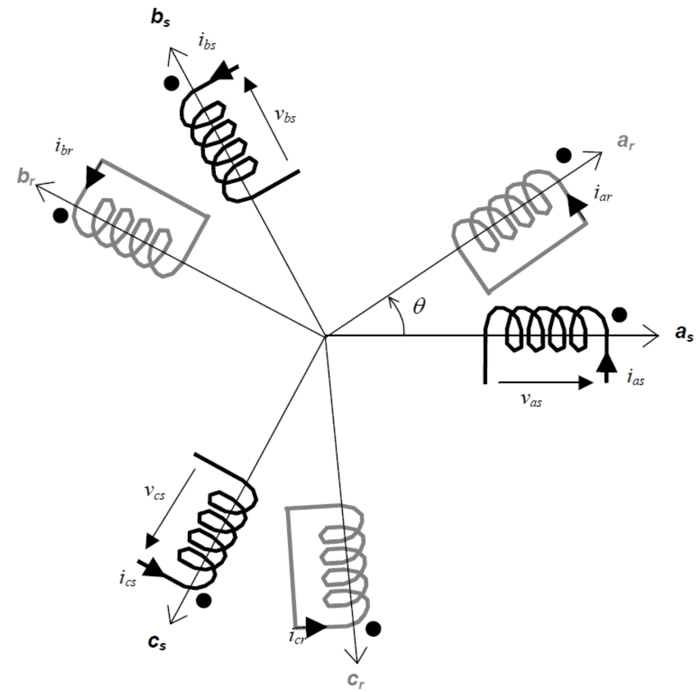
$$m_1 = M\cos(\theta)$$

$$m_2 = M\cos\left(\theta - \frac{2\pi}{3}\right)$$

$$m_3 = M\cos\left(\theta + \frac{2\pi}{3}\right)$$

L_s : is the self-inductance of a stator phase
m_s : the mutual inductance between two stator phases
M : is the maximum of the mutual inductance between a stator phase and a rotor phase

Modelling of Induction machine



Flux equations

φ_{ar} = Self flux + Mutual flux

φ_{ar} = $m_1 i_{as} + m_2 i_{bs} + m_3 i_{cs} + L_r i_{ar} + m_r i_{br} + m_r i_{cr}$

$$m_1 = m_{sr}\cos(\theta)$$

$$m_2 = m_{sr}\cos\left(\theta - \frac{2\pi}{3}\right)$$

$$m_3 = m_{sr}\cos\left(\theta + \frac{2\pi}{3}\right)$$

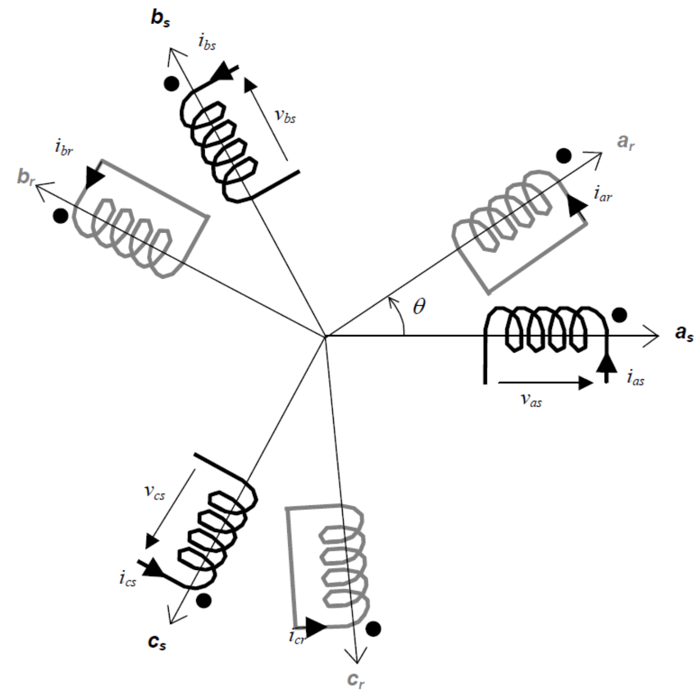
L_r : is the self-inductance of a rotor phase

m_r : the mutual inductance between two rotor phases

m_{sr} : is the maximum of the mutual inductance between a stator phase and a rotor phase

Modelling of Induction machine

Compact presentation



$$V = [v_{as}\ \ v_{bs}\ \ v_{cs}\ \ v_{ar}\ \ v_{br}\ \ v_{cr}]$$

$$I = [I_{as}\ \ I_{bs}\ \ I_{cs}\ \ I_{ar}\ \ I_{br}\ \ I_{cr}]$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \\ v_{ar} \\ v_{br} \\ v_{cr} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \\ I_{ar} \\ I_{br} \\ I_{cr} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} L_s & m_s & m_s & m_1 & m_3 & m_2 \\ m_s & L_s & m_s & m_2 & m_1 & m_3 \\ m_s & m_s & L_s & m_3 & m_2 & m_1 \\ m_1 & m_2 & m_3 & L_r & m_r & m_r \\ m_3 & m_1 & m_2 & m_r & L_r & m_r \\ m_2 & m_3 & m_1 & m_r & m_r & L_r \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \\ I_{ar} \\ I_{br} \\ I_{cr} \end{bmatrix}$$

Modelling of Induction machine

Electromagnetic torque

$$T_e = \frac{p}{2} I^t \frac{dL}{d\theta} I$$

P: is the number of pole pairs

Mechanical equation

$$J \frac{d\omega_m}{dt} = T_e - T_l$$

J: is the moment of inertia
T_l: is the load torque
ω_m: is the speed of the machine

$$J \frac{d\omega_m}{dt} = T_e - T_l - f\omega_m$$

f : is viscous friction

Modelling of Induction machine

$(\boldsymbol{a-b-c})$ **to** $(\boldsymbol{\alpha-\beta})$ **Transformation**

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$(\boldsymbol{\alpha-\beta})\ \boldsymbol{to}\ (\boldsymbol{a-b-c})$ **Transformation**

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

Modelling of Induction machine

Stator variables

v_{sa}, v_{sb}, v_{sc}
I_{sa}, I_{sb}, I_{sc}

Rotor variables

v_{ra}, v_{rb}, v_{rc}
I_{ra}, I_{rb}, I_{rc}

Induction machine model

$$V = RI + \frac{d}{dt}\Phi$$

$$V = RI + \frac{d}{dt}[LI]$$

Induction machine model in $(\boldsymbol{\alpha - \beta})$ frame

$$V_{s\alpha\beta} = CV_{sabc}$$

$$V_{r\alpha\beta} = CV_{rabc}$$

$$V_{\alpha\beta} = \begin{bmatrix} V_{s\alpha\beta} \\ V_{r\alpha\beta} \end{bmatrix}$$

Modelling of Induction machine

Induction machine model in $(\boldsymbol{\alpha - \beta})$ frame

$$I_{s\alpha\beta} = CI_{sabc}$$

$$I_{r\alpha\beta} = CI_{rabc}$$

$$I_{\alpha\beta} = \begin{bmatrix} I_{s\alpha\beta} \\ I_{r\alpha\beta} \end{bmatrix}$$

$$V = RI + \frac{d}{dt}[LI]$$

$$C^{-1}V_{\alpha\beta} = RC^{-1}I_{\alpha\beta} + \frac{d}{dt}[LC^{-1}I_{\alpha\beta}]$$

$$V_{\alpha\beta} = CRC^{-1}I_{\alpha\beta} + C\frac{d}{dt}[LC^{-1}I_{\alpha\beta}]$$

$$V_{\alpha\beta} = CRC^{-1}I_{\alpha\beta} + CLC^{-1}\frac{d}{dt}[I_{\alpha\beta}] + C\frac{d}{dt}[LC^{-1}]I_{\alpha\beta}$$

$$V_{\alpha\beta} = CRC^{-1}I_{\alpha\beta} + CLC^{-1}\frac{d}{dt}[I_{\alpha\beta}] + C\frac{d}{dt}[L]C^{-1}I_{\alpha\beta}$$

Modelling of Induction machine

$$V_{\alpha\beta} = CRC^{-1}I_{\alpha\beta} + CLC^{-1}\frac{d}{dt}[I_{\alpha\beta}] + C\frac{d}{dt}[L]C^{-1}I_{\alpha\beta}$$

$$R = \begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix}, \qquad R_s = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \qquad R_r = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix}$$

$$\begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} R_s & 0 \\ 0 & R_r \end{bmatrix}\begin{bmatrix} C^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix} = \begin{bmatrix} CR_sC^{-1} & 0 \\ 0 & CR_rC^{-1} \end{bmatrix} = R$$

$$V_{\alpha\beta} = RI_{\alpha\beta} + L_{\alpha\beta}\frac{d}{dt}[I_{\alpha\beta}] + G\frac{d\theta}{dt}I_{\alpha\beta}$$

G : is the speed matrix

Modelling of Induction machine

$$V_{\alpha\beta} = RI_{\alpha\beta} + L_{\alpha\beta}\frac{d}{dt}\left[I_{\alpha\beta}\right] + G\frac{d\theta}{dt}I_{\alpha\beta}$$

$$L_{\alpha\beta} = \begin{bmatrix} L_s - m_s & 0 & \frac{3}{2}M\cos(\theta) & -\frac{3}{2}M\sin(\theta) \\ 0 & L_s - m_s & \frac{3}{2}M\sin(\theta) & \frac{3}{2}M\cos(\theta) \\ \frac{3}{2}M\cos(\theta) & \frac{3}{2}M\sin(\theta) & L_r - m_r & 0 \\ -\frac{3}{2}M\sin(\theta) & \frac{3}{2}M\cos(\theta) & 0 & L_r - m_r \end{bmatrix}$$

Modelling of Induction machine

$$V_{\alpha\beta} = RI_{\alpha\beta} + L_{\alpha\beta}\frac{d}{dt}\left[I_{\alpha\beta}\right] + G\frac{d\theta}{dt}I_{\alpha\beta}$$

$$G = \begin{bmatrix} 0 & 0 & -\frac{3}{2}M\sin(\theta) & -\frac{3}{2}M\cos(\theta) \\ 0 & 0 & \frac{3}{2}M\cos(\theta) & -\frac{3}{2}M\sin(\theta) \\ -\frac{3}{2}M\sin(\theta) & \frac{3}{2}M\cos(\theta) & 0 & 0 \\ -\frac{3}{2}M\cos(\theta) & -\frac{3}{2}M\sin(\theta) & 0 & 0 \end{bmatrix}$$

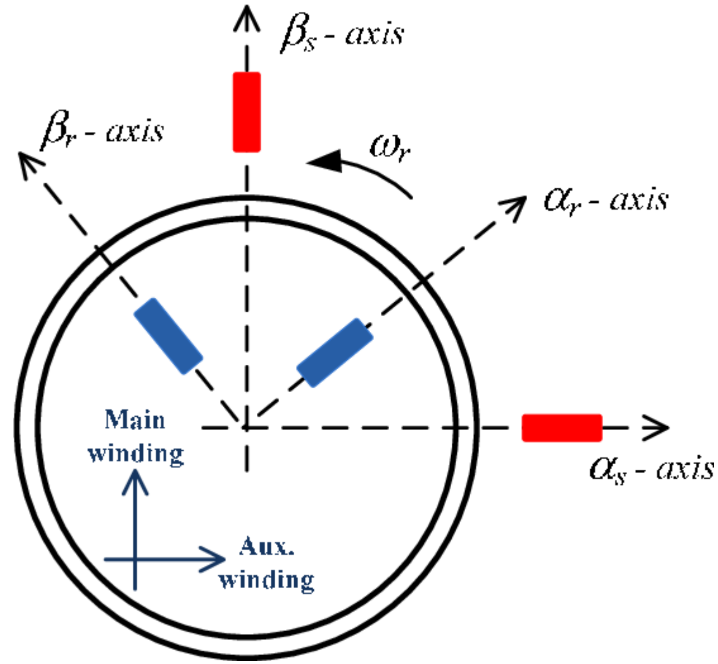
Modelling of Induction machine

$$\begin{bmatrix} i_{\alpha_r} \\ i_{\beta_r} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_{\alpha_r}^s \\ i_{\beta_r}^s \end{bmatrix}$$

$$\begin{bmatrix} i_{\alpha_r}^s \\ i_{\beta_r}^s \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} i_{\alpha_r} \\ i_{\beta_r} \end{bmatrix}$$

$$V_{\alpha\beta} = RI_{\alpha\beta} + L_{\alpha\beta}\frac{d}{dt}[I_{\alpha\beta}] + G\frac{d\theta}{dt}I_{\alpha\beta}$$

$$L_{\alpha\beta} = \begin{bmatrix} L_{ls} - m_s & 0 & \frac{3}{2}M & 0 \\ 0 & L_{ls} - m_s & 0 & \frac{3}{2}M \\ \frac{3}{2}M & 0 & L_{lr} - m_r & 0 \\ 0 & \frac{3}{2}M & 0 & L_{lr} - m_r \end{bmatrix}$$

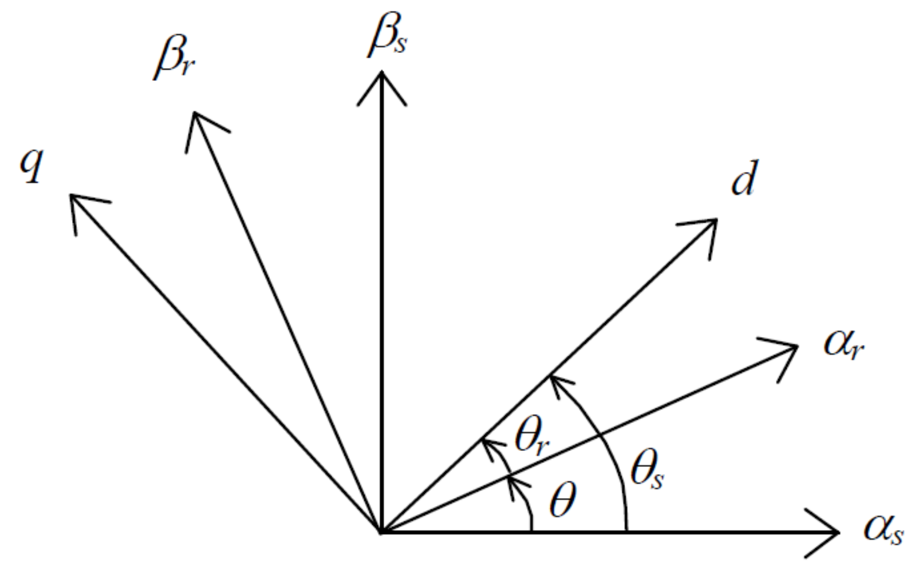


Modelling of Induction machine

<u>**Park transformation**</u>

$$\begin{bmatrix} x_{\alpha s} \\ x_{\beta s} \end{bmatrix} = \begin{bmatrix} \cos(\theta_s) & -\sin(\theta_s) \\ \sin(\theta_s) & \cos(\theta_s) \end{bmatrix} \begin{bmatrix} x_{d_s} \\ x_{q_s} \end{bmatrix}$$

$$\begin{bmatrix} x_{\alpha r} \\ x_{\beta r} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & -\sin(\theta_r) \\ \sin(\theta_r) & \cos(\theta_r) \end{bmatrix} \begin{bmatrix} x_{d_r} \\ x_{q_r} \end{bmatrix}$$



$$\theta_s = \theta + \theta_r$$

$$[v_{dqs}] = R_s[i_{dqs}] + \dot{\theta}_s P\left(\frac{\pi}{2}\right)[\varphi_{dqs}] + \frac{d}{dt}[\varphi_{dqs}]$$

$$[v_{dqr}] = R_r[i_{dqr}] + \dot{\theta}_r P\left(\frac{\pi}{2}\right)[\varphi_{dqr}] + \frac{d}{dt}[\varphi_{dqr}]$$

$$\begin{bmatrix} \varphi_{dqs} \\ \varphi_{dqr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M_{sr} & 0 \\ 0 & L_s & 0 & M_{sr} \\ M_{sr} & 0 & L_r & 0 \\ 0 & M_{sr} & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{dqs} \\ i_{dqr} \end{bmatrix}$$

$$L_s = L_{ls} - m_s$$

$$L_r = L_{lr} - m_r$$

$$M_{sr} = \frac{3}{2} M$$

Modelling of Induction machine

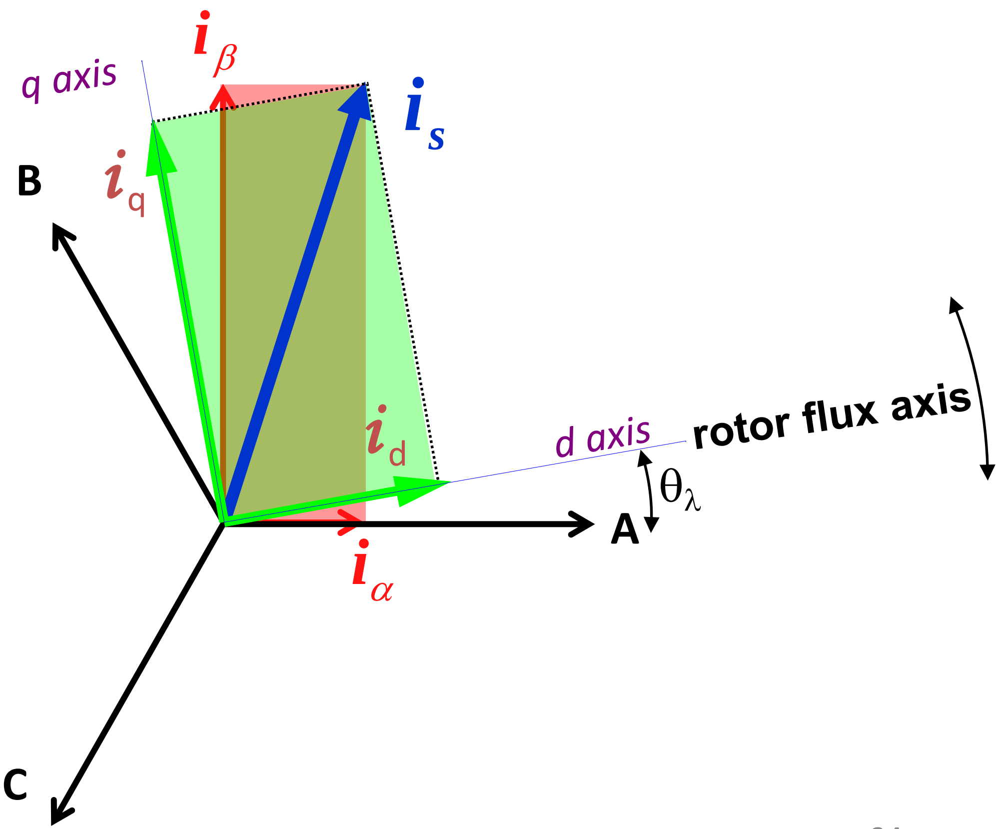
Electromagnetic torque expression

$$T_e = p(\phi_{ds} i_{qs} - \phi_{qs} i_{ds})$$

$$T_e = p(\phi_{qr} i_{dr} - \phi_{dr} i_{qr})$$

$$T_e = pM_{sr}(i_{qs} i_{dr} - i_{ds} i_{qr})$$

$$T_e = p\frac{M_{sr}}{L_r}(\phi_{dr} i_{qs} - \phi_{qr} i_{ds})$$



Modelling of Induction machine

State space model

$$\begin{bmatrix} \frac{di_{sd}}{dt} \\ \frac{di_{sq}}{dt} \\ \frac{d\phi_{rd}}{dt} \\ \frac{d\phi_{rq}}{dt} \end{bmatrix} = \begin{bmatrix} -\gamma & \omega_s & ba & bp\Omega \\ -\omega_s & -\gamma & -bp\Omega & ba \\ aM_{sr} & 0 & -a & (\omega_s - p\Omega) \\ 0 & aM_{sr} & -(\omega_s - p\Omega) & -a \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ \phi_{rd} \\ \phi_{rq} \end{bmatrix} + \begin{bmatrix} m_1 v_{sd} \\ m_1 v_{sq} \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{d\Omega}{dt} = m(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) - c\Omega - \frac{1}{J} T_l$$

Modelling of Induction machine

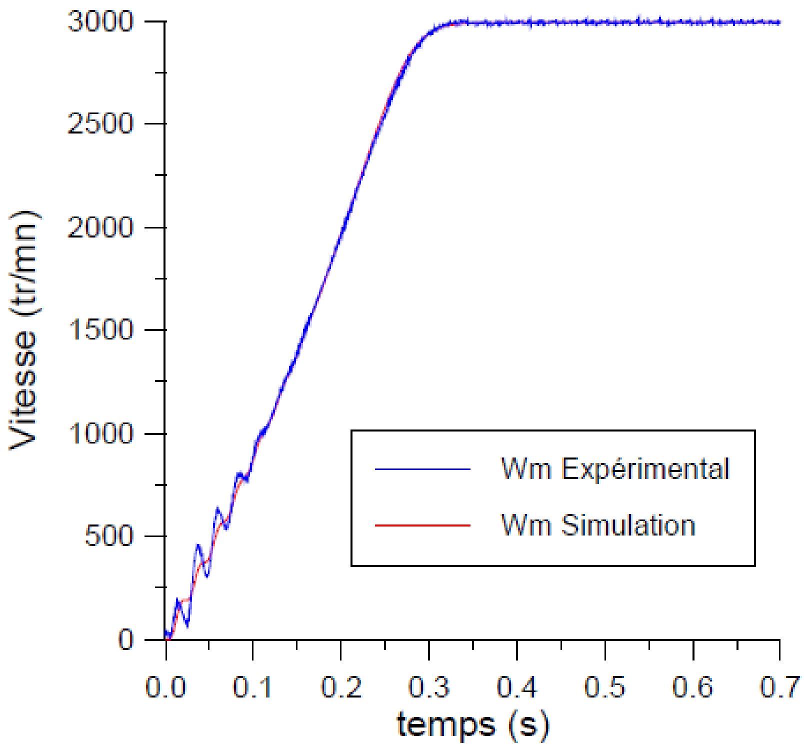
Nonlinear model

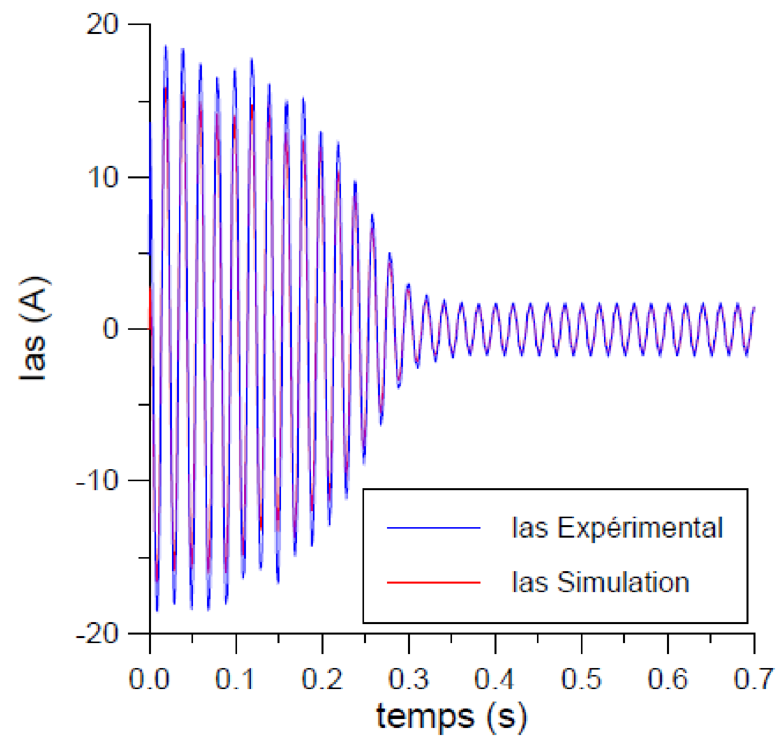
$$
\begin{bmatrix} \frac{di_{sd}}{dt} \\ \frac{di_{sq}}{dt} \\ \frac{d\phi_{rd}}{dt} \\ \frac{d\phi_{rq}}{dt} \\ \frac{d\Omega}{dt} \\ \frac{dT_l}{dt} \end{bmatrix} = \begin{bmatrix} -\gamma i_{sd} + \omega_s i_{sq} + ba\phi_{rd} + bp\Omega\phi_{rq} \\ -\omega_s i_{sd} - \gamma i_{sq} - bp\Omega\phi_{rd} + ba\phi_{rq} \\ aM_{sr} i_{sd} - a\phi_{rd} + (\omega_s - p\Omega)\phi_{rq} \\ aM_{sr} i_{sq} - (\omega_s - p\Omega)\phi_{rd} - a\phi_{rq} \\ m(\phi_{rd} i_{sq} - \phi_{rq} i_{sd}) - c\Omega - \frac{1}{J} T_l \\ 0 \end{bmatrix} + \begin{bmatrix} m_1 & 0 \\ 0 & m_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix}
$$

$$
a = R_r/L_r, \quad b = M_{sr}/\sigma L_s L_r, \quad c = f_v/J, \quad \gamma = (L_r^2 R_s + M_{sr}^2 R_r)/(\sigma L_s L_r^2),
$$

$$
\sigma = 1 - (M_{sr}^2/L_s L_r), \quad m = pM_{sr}/J L_r, \quad m_1 = 1/\sigma L_s .
$$

Modelling of Induction machine





Comparison between simulated model and experimental measurment

Control of the induction machine

Field oriented control of induction machine

- Same principle as the FOC without model
- Stability
- Robustness

- FOC is a torque control
- Several approaches for the FOC

$$T_e = p\frac{M}{L_r}(\phi_{dr} i_{qs} - \phi_{qr} i_{ds})$$

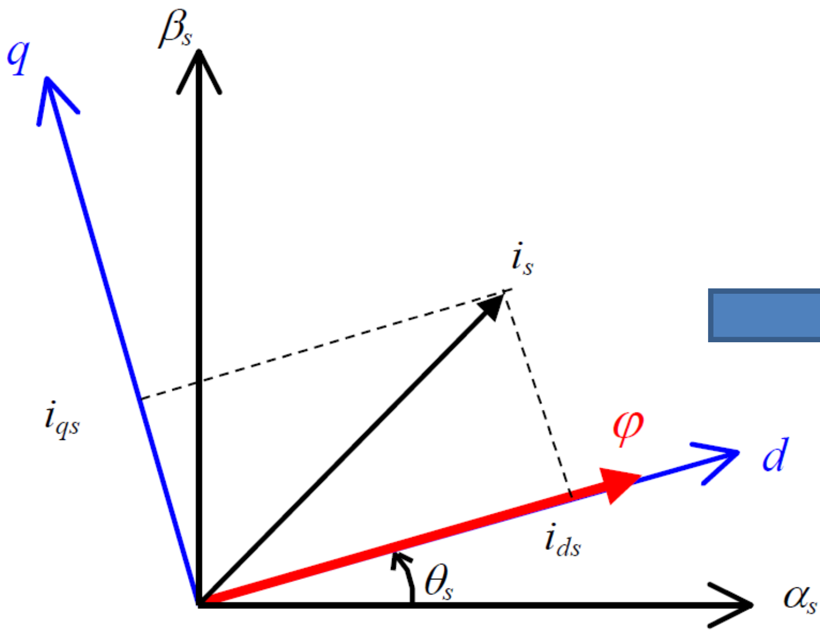
- Control the induction machine as DC machine

$$T_e = kI$$

Control of the induction machine

$$T_e = p\frac{M}{L_r}(\phi_{dr} i_{qs} - \phi_{qr} i_{ds})$$

$$T_e = kI$$



$$T_e = p\frac{M}{L_r}(\phi_{dr} i_{qs} - \cancel{\phi_{qr} i_{ds}})$$

$$T_e = p\frac{M}{L_r}\phi_{dr} i_{qs}$$

Control of the induction machine

$$
\begin{cases}
v_{ds} = R_s i_{ds} - \dot{\theta}_s \varphi_{qs} + \dfrac{d\varphi_{ds}}{dt} \\
v_{qs} = R_s i_{qs} + \dot{\theta}_s \varphi_{ds} + \dfrac{d\varphi_{qs}}{dt} \\
v_{dr} = 0 = R_r i_{dr} - \dot{\theta}_r \varphi_{qr} + \dfrac{d\varphi_{dr}}{dt} \\
v_{qr} = 0 = R_r i_{qr} + \dot{\theta}_r \varphi_{dr} + \dfrac{d\varphi_{qr}}{dt}
\end{cases}
$$

$$
\begin{bmatrix} \varphi_{dqs} \\ \varphi_{dqr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{dqs} \\ i_{dqr} \end{bmatrix}
$$

$$\varphi_r = \varphi_{dr}$$

$$V_{ds} = R_s I_{ds} + \sigma L_s \frac{dI_{ds}}{dt} + \frac{M}{L_r}\frac{d\varphi}{dt} - \omega_s \sigma L_s I_{qs}$$

$$V_{qs} = R_s I_{qs} + \sigma L_s \frac{dI_{qs}}{dt} + \omega_s \frac{M}{L_r}\varphi_r + \omega_s \sigma L_s I_{ds}$$

$$\tau_r \frac{d\varphi_r}{dt} + \varphi_r = M I_{ds}$$

$$\omega_r = \frac{M}{\tau_r \varphi_r} I_{qs}$$

$$C_e = p\frac{M}{L_r}\varphi_r I_{qs}$$

Control of the induction machine

$$V_{ds} = (R_s + p\sigma L_s)I_{ds} + p\frac{M}{L_r}\varphi_r - \omega_s \sigma L_s I_{qs}$$

$$V_{qs} = (R_s + p\sigma L_s)I_{qs} + \omega_s\frac{M}{L_r}\varphi_r + \omega_s \sigma L_s I_{ds}$$

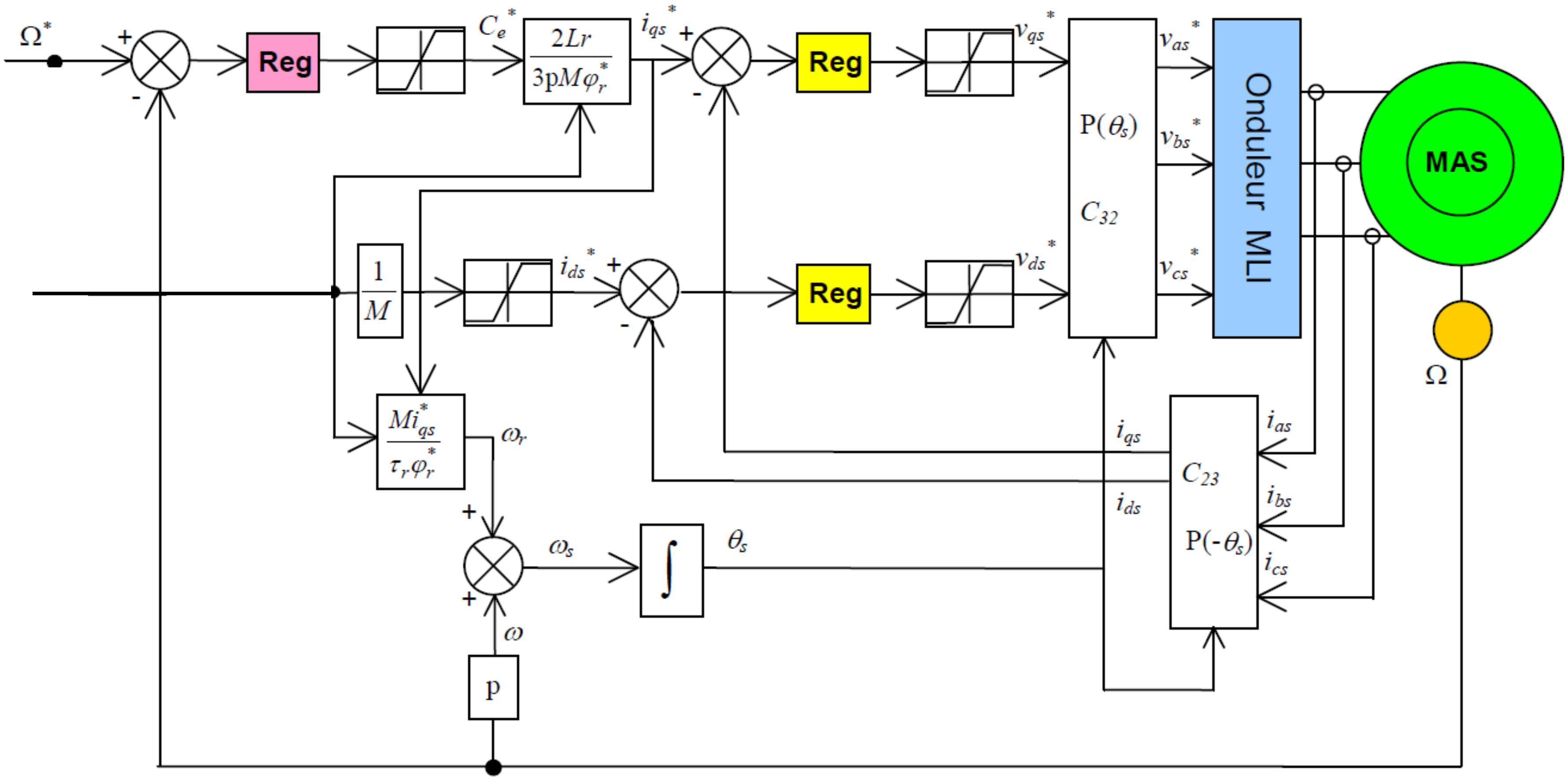
$$\varphi_r = \frac{M}{1 + p\tau_r} I_{ds}$$

$$\omega_r = \frac{M}{\tau_r \varphi_r} I_{qs}$$

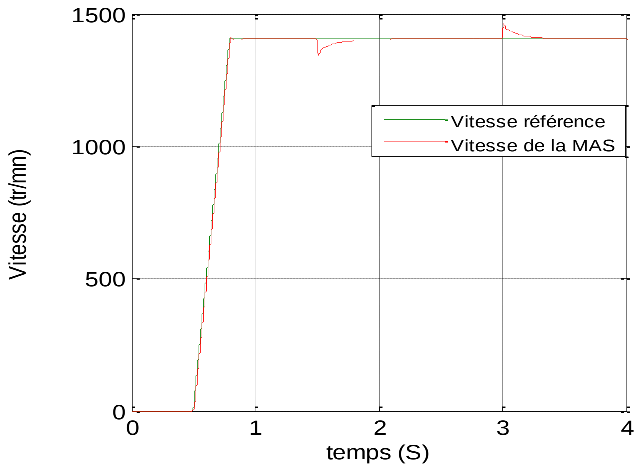
$$\Longrightarrow \quad \varphi_r = M I_{ds}$$

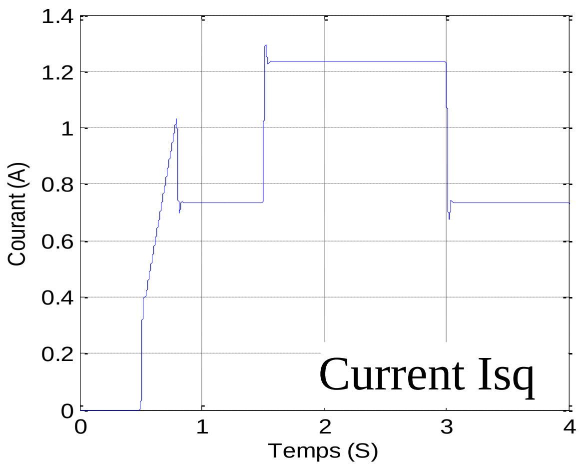
$$\theta_s = \int (p\Omega + \frac{I_{qs}^*}{\tau_r I_{ds}^*})dt \quad \text{où} \quad I_{ds}^* = \frac{\varphi_r^*}{M}$$

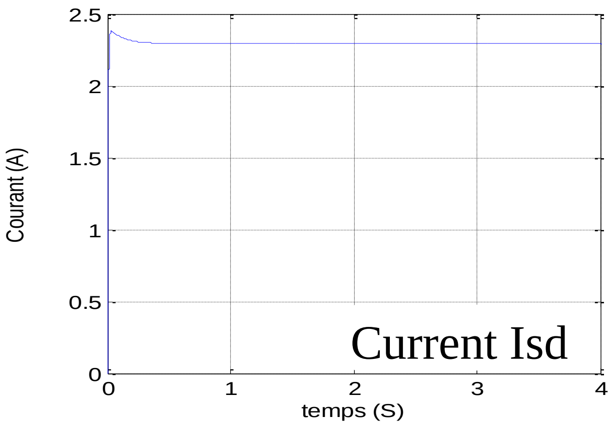
Control of the induction machine

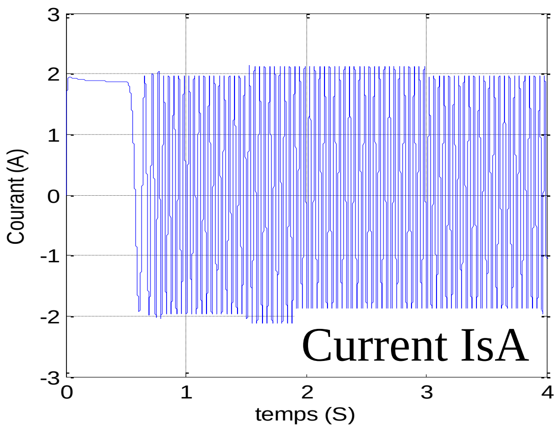


Simulation results









Experimental results