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Reliable fuzzy stabilization against sensor faults

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***Abstract*—Reliable control problems for T-S fuzzy systems with sensor faults are studied in this paper. For a class of fuzzy systems, the premise variables of which are dependent on the system states, a type of new conditions for designing fuzzy reliable controllers are proposed by applying the properties of product inference engines. In contrast to the existing approach, the new conditions are with less conservatism for T-S fuzzy system with partly unmeasurable premise variables. An example is given to show the advantage of the proposed approach.**

I. Introduction

High reliability are necessary in many engineering control systems. In general, malfunction of some hardware or software often corrupt the measurements of the sensors, broken or bad communication, sudden environmental disturbances, which will lead to sensor being partial or complete failure. Then these unreliable sensors can reduce performance or destroy the stability of the systems. Thus, various reliable control techniques are developed for remove the bad influence of sensor faults, see [1] and the reference therein.

The above mentioned conditions are for designing reliable controllers for linear systems. But many engineering systems are nonlinear, then, various reliable control techniques of nonlinear systems are proposed, see [2] and the references therein. In nonlinear control theory, an important approach is to model the considered nonlinear systems as Takagi and Sugeno (T-S) fuzzy models, which are locally linear time-invariant systems connected by IF-THEN rules [3]. Therefore, the conventional linear system theory is applicable for analysis and synthesis of the nonlinear control systems. In particular, reliable control synthesis for nonlinear systems based on T-S fuzzy models received considerable attention in recent years[4], for example, by using multiple Lyapunov functions, the conditions for designing $\mathcal{L}_2/H_\infty$ fuzzy static output feedback reliable controller is proposed in [5]. For stochastic fuzzy systems with sensor and actuator faults, sliding mode observer methods are applicable in [6]. Note that the premise variables may be dependent on system state variables, then the unreliable measurement values for system states will lead to unreliable premise variables. Therefore, the existing fuzzy reliable control techniques might be ineffective for the T-S fuzzy systems[7]. Motivated by this, a type of fuzzy reliable control approaches for T-S fuzzy systems will be developed in this paper.

The paper is organized as follows. Section II gives some notations and system description. A type of new conditions for designing fuzzy reliable controllers are presented in Section III. A numerical example is given to show the effectiveness of the new approaches in Section IV. Concluding remarks are given in Section V.

II. System Description

We consider the following nonlinear system, which is described by the T-S fuzzy system model:

Plant Rule $(i_1 i_2 \cdots i_p)$**:**
IF $\xi_1(t)$ is M_{1i_1} and $\xi_2(t)$ is $M_{2i_2}, \cdots, \xi_p(t)$ is M_{pi_p}
THEN

$$\dot{x}(t) = A_{i_1 i_2 \cdots i_p} x(t) + B_{i_1 i_2 \cdots i_p} u(t) \tag{1}$$

$x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $v(t) = [\xi_1(t)\ \xi_2(t)\ \cdots\ \xi_p(t)\ ]^T \in \mathbb{R}^p$, $\xi_i(t)$, $i = 1, \cdots, p$ are the premise variables, M_{ji_j}, $j = 1, \cdots, p$, $i_j = 1, \cdots, r_j$ is an $\xi_j(t)$-based fuzzy set and they are linguistic terms characterized by fuzzy membership functions $M_{ji_j}(\xi_j(t))$, where r_j be the number of $\xi_j(t)$-based fuzzy sets. Then, the fuzzy weights consists of $r = \prod_{i=1}^{p} r_i$ If-Then rules.

The following T-S fuzzy model (2) can be obtained by using the fuzzy inference method with a singleton fuzzifier, product inference, and center average defuzzifiers.

Let

$$\eta_{ji_j}(\xi_j(t)) = \frac{M_{ji_j}(\xi_j(t))}{\sum_{l_j=1}^{r_j} M_{jl_j}(\xi_j(t))}, \text{for } 1 \le j \le p, 1 \le i_j \le r_j \tag{3}$$

then the fuzzy system can be written as follows:

$$\dot{x}(t) = \sum_{i_1=1}^{r_1} \sum_{i_2=1}^{r_2} \cdots \sum_{i_p=1}^{r_p} \left(\prod_{j=1}^{p} \eta_{ji_j}(\xi_j(t)) \right) \times \left(A_{i_1 i_2 \cdots i_p} x(t) + B_{i_1 i_2 \cdots i_p} u(t) \right) \tag{4}$$

It is resulted from (3) that

$$\sum_{i_j=1}^{r_j} \eta_{ji_j}(\xi_j(t)) = 1, \quad \text{for } 1 \le j \le p \tag{5}$$

In the existing literature, there are various fuzzy control schemes for T-S fuzzy systems, for example, the parallel distributed compensation (PDC) control scheme [3], switched PDC control scheme [8], non-PDC control scheme [9], and so

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$$\dot{x}(t) = \frac{\sum\limits_{i_1=1}^{r_1}\sum\limits_{i_2=1}^{r_2}\cdots\sum\limits_{i_p=1}^{r_p}\left(\prod\limits_{j=1}^{p}M_{ji_j}(\xi_j(t))\right)\left(A_{i_1i_2\cdots i_p}x(t)+B_{i_1i_2\cdots i_p}u(t)\right)}{\sum\limits_{i_1=1}^{r_1}\sum\limits_{i_2=1}^{r_2}\cdots\sum\limits_{i_p=1}^{r_p}\prod\limits_{j=1}^{p}M_{ji_j}(\xi_j(t))} \quad (2)$$

on, where the PDC control scheme is often used and it is also applied in this paper.

Control Rule $(i_1i_2\cdots i_p)$**:**

IF $\xi_1(t)$ is M_{1i_1} and $\xi_2(t)$ is $M_{2i_2},\cdots,\xi_p(t)$ is M_{pi_p}

THEN $u(t)=K_{i_1i_2\cdots i_p}x(t)$ (6)

Then the global fuzzy controller is obtained as:

$$u(t)=\sum_{i_1=1}^{r_1}\sum_{i_2=1}^{r_2}\cdots\sum_{i_p=1}^{r_p}\prod_{j=1}^{p}\eta_{ji_j}(\xi_j(t))K_{i_1i_2\cdots i_p}x(t) \quad (7)$$

From (7) and (4), we can obtain the closed-loop system as follows:

$$\dot{x}(t)=\sum_{i_1=1}^{r_1}\sum_{i_2=1}^{r_2}\cdots\sum_{i_p=1}^{r_p}\sum_{l_1=1}^{r_1}\cdots\sum_{l_p=1}^{r_p}\prod_{j=1}^{p}\eta_{ji_j}(\xi_j(t))\eta_{jl_j}(\xi_j(t))$$
$$\times(A_{i_1i_2\cdots i_p}+B_{i_1i_2\cdots i_p}K_{l_1l_2\cdots l_p})x(t) \quad (8)$$

A type of new descriptions by using set theory for T-S fuzzy systems will be given for exploiting new reliable control methods.

We define the following set

$$\Omega_i=\{1,2,\cdots,r_i\} \qquad i=1,2\cdots,p \quad (9)$$

Denote the product of the sets Ω_i, $i=1,\,2,\,\cdots,\,p$ as

$$\Omega_1\times\Omega_2\times\cdots\times\Omega_p$$
$$=\prod_{i=1}^{p}\Omega_i=\{i_1i_2\cdots i_p:\ i_1\in\Omega_1,i_2\in\Omega_2,\cdots,i_p\in\Omega_p\}$$

Then the closed-loop system (8) can also be described as:

$$\dot{x}(t)=\sum_{\varpi\in\prod_{i=1}^{p}\Omega_i}\sum_{\rho\in\prod_{i=1}^{p}\Omega_i}\eta_\varpi\eta_\rho(A_\varpi+B_\varpi K_\rho)x(t) \quad (10)$$

where

$$\eta_\varpi=\prod_{j=1}^{p}\eta_{j\varpi_{\langle j]}}(\xi_j(t)),\varpi=\varpi_{\langle 1]}\varpi_{\langle 2]}\cdots\varpi_{\langle p]},\varpi\in\prod_{i=1}^{p}\Omega_i$$
$$\eta_\rho=\prod_{j=1}^{p}\eta_{j\rho_{\langle j]}}(\xi_j(t)),\rho=\rho_{\langle 1]}\rho_{\langle 2]}\cdots\rho_{\langle p]},\rho\in\prod_{i=1}^{p}\Omega_i \quad (11)$$

In this paper, assume that only a multiplicative sensor fault occurs, which is described by the following equation.

Definition 1: [5] (Sensor multiplicative fault) The sensor for measuring a variable $\xi(t)\in R$ is said to have fault at time $T_f>0$, if the output of the sensor

$$\xi^F(t)=f(t)\xi(t),\quad 0\le f(t)<1,\quad \forall t>T_f \quad (12)$$

A part of premise variables in the controller (7) are often dependent on the states. If one sensor for measuring some state is failed, then the premise variables dependent on the state and the state itself, which are used in the fuzzy controller, are both unreliable.

For instance, if some sensor fault leads to the feedback state x_q and the premise variables ξ_{m_1}, ξ_{m_2}, $\cdots$, ξ_{m_s} of the fuzzy controller being unreliable, then the fuzzy controller with the sensor fault can be described as follows:

$$u(t)=\sum_{i_1=1}^{r_1}\sum_{i_2=1}^{r_2}\cdots\sum_{i_p=1}^{r_p}\left(\prod_{j=1,j\neq m_1,\cdots,m_s}^{p}\eta_{ji_j}(\xi_j(t))\right)\times$$
$$\left(\eta_{m_1i_{m_1}}(v_{m_1}^F(t))\cdots\eta_{m_si_{m_s}}(v_{m_s}^F(t))\right)K_{i_1i_2\cdots i_p}F_qx(t) \quad (13)$$

where

$$F_m=diag\left[1,\cdots,1,f_m,1,\cdots,1\right]_{n\times n},0\le f_m\le 1 \quad (14)$$

and $v_{m_1}^F(t)$, $\cdots$, $v_{m_s}^F(t)$ are the unreliable premise variables in the fuzzy controller.

III. Main result

In this section, we will propose a type of reliable controller design conditions for T-S fuzzy systems with sensor faults. The subcripts of all premise variables are collected as some special sets:

$$\Lambda=\{1,2,\cdots,p\}$$
$$\Lambda_i=\{\chi|\text{ the premise variable }\xi_\chi\text{ is dependent on }x_i\}$$

For the fuzzy systems (1), if one sensor for measuring x_q has fault, then for $i\in\Lambda-\Lambda_q$, $\xi_i(t)$ is reliable, but for $i\in\Lambda_q$, $\xi_i(t)$ is unreliable. Then the fuzzy controller with the sensor variable fault is described as follows:

$$u(t)=\sum_{i_1=1}^{r_1}\sum_{i_2=1}^{r_2}\cdots\sum_{i_p=1}^{r_p}\left(\prod_{j\in\Lambda-\Lambda_q}\eta_{ji_j}(\xi_j(t))\right)\times$$
$$\left(\prod_{j\in\Lambda_q}\eta_{ji_j}(\xi_j^F(t))\right)K_{i_1i_2\cdots i_p}F_qx(t)$$
$$=\sum_{\rho\in\prod_{i=1}^{p}\Omega_i}\left(\prod_{j\in\Lambda-\Lambda_q}\eta_{j\rho_{\langle j]}}(\xi_j(t))\right)\times$$
$$\left(\prod_{j\in\Lambda_q}\eta_{j\rho_{\langle j]}}(\xi_j^F(t))\right)K_\rho F_qx(t)$$

Consider the following binary relation on Ω_i^2,

$$\mathbb{R}_i=\{(i_1i_2,j_1j_2):(i_1=j_1\text{ and }i_2=j_2),$$

$$\text{or } (i_1 = j_2 \text{ and } i_2 = j_1), i_1 i_2, j_1 j_2 \in \Omega_i^2\} \quad (15)$$

It is easily shown that the relation $\mathbb{R}_i$ is an equivalence relation on the set Ω_i^2, then by set theory, the set $\Omega_i^2/\mathbb{R}_i = \{[\![\mathrm{x}]\!]_{\mathbb{R}} : \mathrm{x} \in \Omega_i^2\}$ is a partition of Ω_i^2.

Theorem 1: If there exist matrices $Q_q = Q_q^T > 0$, $1 \leq q \leq n$, L_ϖ, $\varpi \in \Omega$, a positive scalar ϵ, such that the following inequalities hold

$$\sum_{\substack{\varpi_{\langle i]}\rho_{\langle i]} \in \mathbb{X}_i \\ i \in \Lambda - \Lambda_q}} \underline{\Phi}_{\varpi\rho q} < 0, \text{ for } \varpi, \rho \in \prod_{j=1}^{p} \Omega_j, \mathbb{X}_1 \in \Omega_1^2/\mathbb{R}_1, \cdots, \mathbb{X}_p \in \Omega_p^2/\mathbb{R}_p, 1 \leq q \leq n \quad (16)$$

$$\sum_{\substack{\varpi_{\langle i]}\rho_{\langle i]} \in \mathbb{X}_i \\ i \in \Lambda - \Lambda_q}} \bar{\Phi}_{\varpi\rho q} < 0, \quad \text{for } \varpi, \rho \in \prod_{j=1}^{p} \Omega_j, \mathbb{X}_1 \in \Omega_1^2/\mathbb{R}_1, \cdots, \mathbb{X}_p \in \Omega_p^2/\mathbb{R}_p, 1 \leq q \leq n \quad (17)$$

where

$$\underline{\Phi}_{\varpi\rho q} = \begin{bmatrix} \mathtt{He}(A_\varpi Q_q + B_\varpi L_\rho) & * \\ Q_q - H + \epsilon L_\rho^T B_\varpi^T & -\epsilon H - \epsilon H^T \end{bmatrix},$$
$$\bar{\Phi}_{\varpi\rho q} = \begin{bmatrix} \mathtt{He}(A_\varpi Q_q + B_\varpi L_\rho) & * \\ \bar{F}_q Q_q - H + \epsilon L_\rho^T B_\varpi^T & -\epsilon H - \epsilon H^T \end{bmatrix},$$
$$\bar{F}_q = \begin{bmatrix} 1, \cdots, 1, 0, 1, \cdots, 1 \end{bmatrix},$$

then the closed-loop system (4) with (6) and $K_\varpi = L_\varpi H^{-1}$, $\varpi \in \Omega$ is asymptotically stable in the normal case and only one sensor failure cases.

Proof: Due to space, the proof is omitted. ■

Remark 1: Note that a set of LMIs with a line search over a scalar variable ϵ are given Theorem 1, therefore the condition of Theorem 1 is unconvex. But ϵ is a scalar variable, a constructive numerical procedure can be given and refer to[10], [11].

IV. Example

An example is given to show the validity of the proposed approach in this section. The following fuzzy model is considered.

Plant Rule $(i_1 i_2)$**:**

IF $\xi_1(t)$ is M_{1i_1} and $\xi_2(t)$ is M_{2i_2}
THEN $\dot{x}(t) = A_{i_1 i_2} x(t) + B_{i_1 i_2} u(t)$

where

$$A_{11} = \begin{bmatrix} 0 & 1 & -3 \\ -1 & -0.7 & -1 \\ 2 & 1 & -4.6 \end{bmatrix}, A_{12} = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0.3 & -1 \\ 2 & 1 & -4.6 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0 & 1 & -3 \\ -1 & -0.7 & -1 \\ 2 & 1 & -1.4 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 & -3 \\ -1 & 0.3 & -1 \\ 2 & 1 & -1.4 \end{bmatrix}$$
$$B_{11} = B_{12} = B_{21} = B_{22} = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix}^T$$

and the fuzzy membership functions are $\eta_{11}(x_1(t)) = 0.5 - 0.5\sin(x_1(t))$, $\eta_{12}(x_1(t)) = 0.5 + 0.5\sin(x_1(t))$, $\eta_{21}(x_2(t)) = 0.5 - 0.5\sin(x_2(t))$, $\eta_{22}(x_2(t)) = 0.5 + 0.5\sin(x_2(t))$. By applying the product inference engine, we can obtain that the weights of the fuzzy rules (11), (12), (21), (22) are $\eta_{11}(x_1)\eta_{21}(x_2)$, $\eta_{11}(x_1)\eta_{22}(x_2)$, $\eta_{12}(x_1)\eta_{21}(x_2)$ and $\eta_{12}(x_1)\eta_{22}(x_2)$.

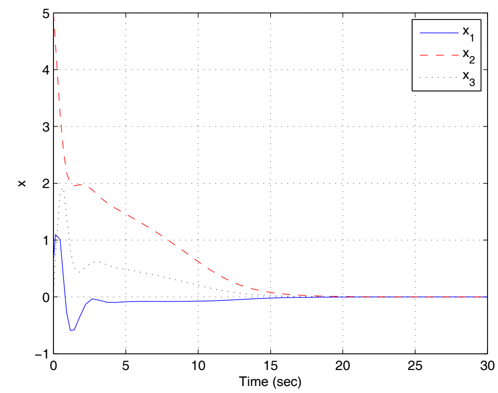

Fig. 1: The responds of the state by Theorem 1

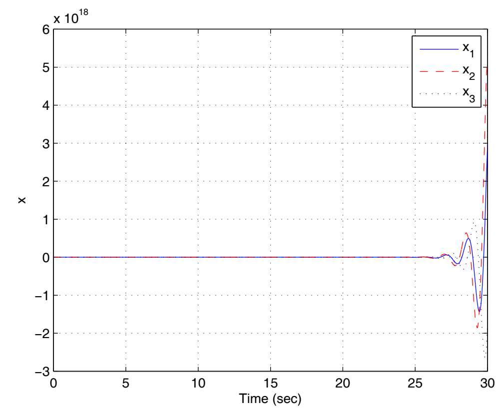

Fig. 2: The responds of the state by the approach in [12]

The conditions of [5], [12] and Theorem 1 are applicable for the example and the obtained results are shown in Table I. When the sensor for measuring state x_2 is outage, the measured value of the state x_2 is 0. Therefore, for the case, $\eta_{21}(x_2^F) = \frac{1+\cos(x_2^F)}{2} = 1$, $\eta_{22}(x_2^F) = \frac{1-\cos(x_2^F)}{2} = 0$, then the fuzzy controller with the sensor fault can be written as follows:

$$\begin{aligned} &u(t) \\ =&(\eta_{11}(x_1)\eta_{21}(x_2^F)K_{11} + \eta_{11}(x_1)\eta_{22}(x_2^F)K_{12} + \eta_{12}(x_1) \\ &\times \eta_{21}(x_2^F)K_{21} + \eta_{12}(x_1)\eta_{22}(x_2^F)K_{22}) \begin{bmatrix} x_1 & x_2^F & x_3 \end{bmatrix}^T \\ =&(\eta_{11}(x_1)\eta_{21}(x_2^F)K_{11} + \eta_{11}(x_1)\eta_{22}(x_2^F)K_{12} \\ &+ \eta_{12}(x_1)\eta_{21}(x_2^F)K_{21} + \eta_{12}(x_1)\eta_{22}(x_2^F)K_{22})F_2 x(t) \end{aligned}$$

The state responds are illustrated in Figs. 1-3.

It can be seen from Figs. 1-3 that the methods of [5] and [12] cannot guarantee the stability of the closed-loop system, but the controller obtained by the condition in Theorem 1

TABLE I: Controller gains

Theorem 1 with $\epsilon = 0.5$	The approach in [5] with $\lambda = 49$	The approach in [12]
$K_{11} = [-0.7509\ 2.0039\ 0.1614]$	$K_1 = [-0.7313\ 11.0063\ -2.4133]$	$K_1 = [1.9659\ 6.5127\ 7.3498]$
$K_{12} = [-0.7621\ 2.2249\ 0.1225]$	$K_2 = [-0.8635\ 6.5585\ 0.8662]$	$K_2 = [2.1216\ 8.0581\ 8.3103]$
$K_{21} = [-0.7760\ 1.9816\ 0.2541]$	$K_3 = [-0.6861\ 4.0168\ -2.3228]$	$K_3 = [1.4487\ 3.0410\ 2.5597]$
$K_{22} = [-0.7861\ 2.1502\ 0.2124]$	$K_4 = [-0.8486\ 1.7740\ 0.1109]$	$K_4 = [1.6044\ 4.5864\ 3.5203]$

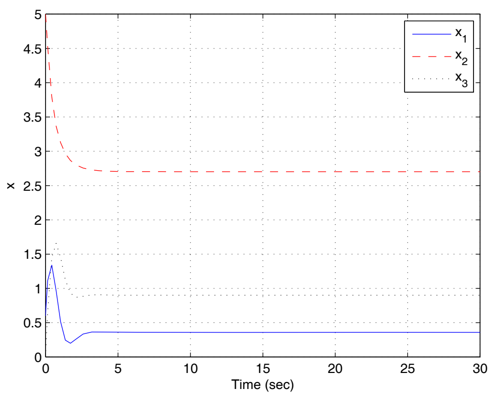

Fig. 3: The responds of the state by the approach in [5]

guarantees the stability. Note that the condition of [5] is for designing reliable controllers, however it is failed for the example, because it is only applicable for the fuzzy system with the premise variables independent on the states.

V. Conclusion

A new condition for designing reliable controller of T-S fuzzy systems with sensor multiplicative faults has been given in this paper. The properties of fuzzy product inference engine is used for obtaining the new condition. In particular, the influences of sensor faults on both the premise variables and system states of the fuzzy controllers are considered, and the proposed controllers can guarantee the stability and system performance when all sensors are reliable as well as when one sensor has fault. A numerical example has been presented to show the effectiveness of the new approach.

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